The Prisoner's Dilemma is an Innovative Tool Valid in the Most Varied Situations: some Applications

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Abstract
The paper was born from the intent to highlight the importance of the prisoner's dilemma and its applications. Prisoner's Dilemma is a comprehensive information game proposed in the 1950s by Albert Tucker as a game theory problem. In addition to being extensively studied in this context, the "dilemma" is also quite well known to the non-technical public as an example of a paradox. John Von Neumann, creator of game theory, was also interested in this dilemma, presented to him by the two creators while working at RAND in 1948. In this article, we have applied the prisoner's dilemma to three very different events: the writing of the thesis, its application in the tobacco manufacturers and the role of pricing policies and the prisoner's dilemma.

Keywords: prisoner's dilemma, history, applications

1. The prisoner's dilemma

Perhaps the best known game is the so-called prisoner's dilemma in which the winnings, so to speak, are the years in prison. The prisoner's dilemma is a well-known non-cooperative game used to represent particular economic situations in which the rational behavior of the players, aimed at maximizing their payoff, leads to a Pareto-inefficient result. Non-cooperation, in fact, is for both players a strictly dominant strategy with respect to cooperation, but it determines a worse situation than that which would have occurred if both had cooperated. The prisoner's dilemma has peculiar characteristics regarding payoffs, players, game duration and the information structure of the game; they are examined in detail as they prove to be important for understanding the game itself. The conception of the prisoner's dilemma is attributed to Tucker. At the basis of this there is what Rasmusen wrote in his text. He says that Tucker was called to give a lecture in game theory at the Department of Psychology in Standford and, precisely for this event, to better explain the ways of applying game theory, he invented a story in line with the particular matrix 2x2, already built by Dresher and Flood. A more precise story about the birth of the Prisoner's Dilemma and the paternity due to Tucker is derived from a conversation between Basu and Harold Kuhn. Basu in his text says that Tucker, on a visit to the Department of Mathematics of Stanford University, for reasons of space received an office at the Department of Psychology. Scholars from the Psychology Department, intrigued by the research Tucker was doing, invited him to give a lecture. Tucker wanted to show them how a balance in dominant strategies was sub-optimal. To this end, he uses a matrix already known in the 1950s, constructing a story that justifies mutual non-cooperation and, therefore, a Pareto-inefficient situation. The story is as follows (told of course using game theory terms): The police arrested two individuals who are seriously suspected of having committed a crime, and who in fact actually committed it. Since the police do not have enough evidence to convict them and must therefore release the suspects unless one of them confesses the crime committed together with the other. The two are kept in separate cells and each receives the following offer from the police: you should report your accomplice and in exchange you will get a penalty discount. The Police also clarifies to the two defendants that if both had chosen the path of non-implication (collaboration) then both would have been stopped pending trial for the maximum time allowed by law. If, on the other hand, one of the two had implicated (collaborated) then he would have been released and at the same time the Police undertook to convince the judge to assign the maximum penalty foreseen by law to the second accused. If, on the other hand, both had implicated (collaborated) they would have gone to prison but would have obtained a discount on the sentence. Therefore, the game that the Police presents to the two players is of the non-cooperative type, with non-zero sum and has the following payoff matrix:

		I player	
		Don't Imply	Imply
II player	Don't Imply	x,x	z,y
	Imply	y,z	w,w

This story suggests that, in terms of expected benefits, the four possible outcomes of the game can be ordered by each prisoner in descending order:
1 denounce without being implicated;
2 do not imply or be implied;
3 to imply and to be implied
4 not to denounce (not to imply) but to be implicated.

This game is characterized by a single Nash equilibrium which requires both defendants to confess the crime. In fact, the confess (imply) strategy strictly dominates the proclaiming (not implying) strategy for both players. In other words, for any strategy chosen by one of the two suspects, the other should in any case confess (imply). The essential feature of this game is that if the players cooperated by declaring themselves innocent they would get a better result than playing the equilibrium strategy. However, since the two defendants cannot communicate with each other and establish binding agreements, the cooperative behavior is not individually rational. In fact, if the cooperative strategy were played by one of the two players, it would be better for the other to play non-cooperatively since by doing so he could achieve a significantly higher payoff than what he would get by responding with a cooperation strategy. The prisoner's dilemma, therefore, highlights a gap between individual rationality and social rationality. The

Nash equilibrium (not to imply / not to imply), in fact, is a decidedly inefficient equilibrium since both players could improve their results by choosing the cooperation strategy. This choice, however, as it is evaluated by the single individual, is too risky and this pushes the two players to solve the problem in terms of "minimax", that is to say with the adoption of the strategy that guarantees the best individual result. in the most favorable hypothesis. In conclusion, it should be noted that each player, fearing that the other confesses, finds that non-cooperation (implication) is a strictly dominant strategy over cooperation (not implying), regardless of the strategy chosen by the other. In support of what is highlighted above, there is the affirmation of Aumann, who in his writing emphasizes that mutual non-cooperation emerges precisely because it is a strictly dominant strategy and not because any communication prior to the start of the game is prohibited, highlighting in this way that the communication aspect is secondary to the impossibility of concluding binding agreements.

2. Some applications of the prisoner's dilemma

2.1 The prisoner's dilemma applied to the degree thesis

In this paragraph we will examine two applications of the prisoner's dilemma to two clearly different situations, let's see them in detail. A university professor, after reading the theses presented by two undergraduates, suspected that the latter have hired a third person (probably a scholar of the subject we will call Dr. Z) to write the thesis for theirs. The Dean of the Faculty, on the advice of the teacher, sets up a commission with the task of dealing with the problem. The commission, after careful consideration, decide to take the following course of action. The two students' rooms are summoned separately so that they cannot each other, after which the commission submits to each graduate a collaboration proposal consisting of the following:

- ✓ If the undergraduates both confess that the thesis was written by Dr. Z, they will be sentenced to a sentence that is not too severe. That is, they will have to work another year to completely rewrite the thesis and the average with which they will present themselves before the Graduation Commission will be decreased by 3 points.
- ✓ If neither of the two undergraduates confesses and both proclaim themselves innocent, they will be able to graduate, but this will happen only after the teacher is convinced that they have mastered the arguments contained in the thesis with confidence. This means that the two undergraduates will be able to graduate in the next session after four months of particularly busy study.
- ✓ If one of the two confesses, while the other insists on proclaiming himself innocent, the first is given a light sentence (he can graduate immediately with an average decreased by 2 points), while the second, who is sensationally exposed while proclaiming himself innocent, will be sentenced to a severe sentence (for example, he will be able to graduate only after a suspension period of 3 years).

What behavior will the two undergraduates adhere to? They can adopt two strategies, confess "C" or proclaim themselves innocent "I", and they will have to choose "C" or "I" without being able to communicate with each other and therefore without being able to coordinate their strategies. The game, consequently, is in simultaneous moves and the payoff matrix expressed in

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terms of utilities assigned to the possible outcomes of the game is illustrated in the following figure:

	C	I
C	0 0	- 6 8
I	8 - 6	3 3

The game is characterized by a single Nash equilibrium which requires both undergraduates to confess. The "confess" strategy strictly dominates the "proclaiming innocence" strategy for both players. In other words, for any strategy chosen by one of the two undergraduates, the other undergraduate should in any case confess. The essential feature of this game is that if the players cooperated and both declared innocent they would get a better result than playing the equilibrium strategy. However, since undergraduates cannot communicate and establish binding agreements, cooperative behavior is not individually rational. In fact, if the cooperative strategy were played by one of the two players, it would be better for the other to play non-cooperatively since by doing so he could achieve a significantly higher payoff than what he would get by responding with a cooperation strategy. The prisoner's dilemma highlights a gap between individual rationality and social rationality. The Nash equilibrium (C, C), in fact, is a decidedly inefficient equilibrium since both players could improve their earnings by choosing the cooperative strategy. This choice, however, as it is evaluated by the individual, is too risky and this pushes the two players to solve the problem in terms of "maximin", ie with the adoption of that strategy that guarantees the best individual result in the most unfavorable hypothesis. From what has been said above, the following aspects emerge that characterize non-cooperative gaming: the impossibility for players to communicate and conclude binding agreements that are susceptible to forced implementation.
It is good to specify for clarity that the conclusions we have reached depend on the hypothesis that the two players cannot establish binding agreements before starting to play.

2.2 The prisoner's dilemma and the tobacco manufacturers
The possibility of entering into binding agreements completely changes the characteristics of the game. In this regard Gadner reports the case of the large US tobacco manufacturers. Before 1964, these companies invested enormous sums to promote cigarette sales with TV commercials. After 1964, the federal government, considering the irrefutable evidence of the damage to health caused by smoking, reached an agreement with the tobacco manufacturers under which the federal government would give up taking legal action against the producers provided that the latter gave up. television advertising and put a sign on cigarette packets that warned consumers

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of the harm caused by smoking. The companies, which expected the agreement to negatively affect their balance sheets, saw, on the contrary, a strong growth in profits. This result, however, is hardly surprising. The competition in the US cigarette market, prior to the agreement, was typically that of the prisoner's dilemma. Companies, for fear of losing market share to their competitors, having adopted a strategy of escalating investments in television advertising which led to a reduction in margins per unit of product without resulting in significant changes in the distribution of market shares. The agreement completely changed the competitive scenario forcing companies to stick to the cooperative strategy which had a largely positive effect on the company's economic results. The prisoner's dilemma arises in many contexts. In the consumer food market, the quantities sold depend significantly on the availability and visibility of the product on supermarket shelves. To convince distribution chains to expand the space granted to their products, companies can be pushed to pursue commercial policies based on strong discounts granted to distributors. Let's assume that this is precisely the case of two companies (Arcofiorito and Mediterraneo) that produce various types of durum wheat pasta. A significant share of the turnover of these companies depends on the conditions of access to the shelves of supermarkets and hypermarkets. Companies can follow two strategies towards distributors. The first consists in practicing a fixed price, the second in practicing strong commercial discounts. The following figure reasonably represents competitive interdependence and reports the levels of utility that the two companies achieve in correspondence with each possible outcome of the competitive game.

	Fixed price	**Heavy discounts**
Fixed price	6 6	10 - 4
Heavy discounts	- 4 10	0 0

It is evident that this game, if it is played only once, admits a single solution, the one in which both Arcofiorito and Mediterraneo offer strong commercial discounts to large retailers. The "fixed price" strategy is extremely risky, because, if the competitor does not respond with the same type of action, it leads to decidedly negative results. Arcofiorito and Mediterraneo are perfectly aware of the fact that, if they jointly practiced fixed prices, they could both obtain substantial advantages. However, due to the heated rivalry that has always existed between Arcofiorito and the Mediterranean, the two companies are unable to coordinate their respective commercial policies. Of course, large retailers take full advantage of the rivalry between the two companies. The example set out above is just one of the application examples of the prisoner's dilemma.

2.3 Pricing policies and the prisoner's dilemma

The literature relating to the prisoner's dilemma has grown considerably since Luce and Raiffa presented it in the standard, symmetrical (two-sides), two-player and one-shot form, although they stressed the possibility that cooperation could emerge from the repetition of the one-period game a very high number of times. Perhaps the best way to tackle the analysis of the repeated prisoner's dilemma is to consider the following example, which was built on the basis of the appropriate developments in the literature on repeated games. Let's assume that we have two companies, the company Boniello S.p.A and the company Rossi S.p.A that produce the same goods. They must decide to adopt either a fixed price or deep discount policy to be competitive on the market. It is unreasonable to assume that the Boniello vs Rossi game is played only once. The competitive interdependence between the two companies is destined to last over time, so it is not possible to establish a precise limit on the number of possible repetitions of the game. If we assume that the game is repeated indefinitely, then cooperative strategy becomes a viable strategy. In this situation, in fact, the cooperative strategy prevails over the non onecooperative only if:$\frac{6}{1-\delta} > 6 \cdot \frac{1-\delta^n}{1-\delta} + 10 \cdot \delta^n + 0 \cdot \frac{1}{1-\delta}$, where the first term is the utility of the strategy that employs cooperative behavior at each stage of the game, the stages are infinite. The second is the waiting utility of the strategy that adopts cooperative moves in the first n stages of the game and that defeats in the n + 1 term stage. By developing the expression, we get$0 > \delta^n \cdot (4 - 10 \cdot \delta)$ which is true only if$(4 - 10 \cdot \delta) < 0$ and therefore only if$\delta > \frac{2}{5}$.Since $\delta = \frac{1}{1+i}$the condition for the cooperative strategy to prevail over the non-cooperative one is that the rate of interest with which future profits are discounted is$i < \frac{3}{2}$. As you can see, for the interest rates entirely reasonable, the cooperative strategy strictly dominates the non-cooperative strategy and it constitutes a Nash equilibrium for the game of the Boniello society against the Rossi society. It is worth pointing out that the condition$i < \frac{3}{2}$ depends on the payoff values entered in the matrix. It is evident that as the relationship between the utility assigned to the outcome that derives from non-cooperative behavior increases compared to that deriving from cooperative behavior, the condition becomes more and more stringent. For example if this ratio were 4 to 1, the condition would become:$i < \frac{1}{3}$ and it is easy to verify that if the ratio became 10 to 1, an interest rate higher than that of 11.11% would be sufficient to make it convenient to withdraw from the cooperative strategy. It is also important to emphasize that the cooperation strategy we have just discussed is not the only balance allowed by the game. There are actually many others. Suppose that the company Boniello S.p.A. announces that it wants to implement a strategy that alternates the fixed price policy with that of commercial discounts if and only if Rossi S.p.A. will always practice the fixed price policy. The condition set by the company Boniello S.p.A. is irreversible, in fact, if by hypothesis the Rossi S.p.A. company should also start practicing the commercial discount policy, then the Boniello S.p.A. company he reiterates that from that precise moment on he will always implement the commercial discount policy. Under what conditions is it convenient for Rossi S.p.A. adhere to this strategy profile? We see. In the meantime, let's start by observing that if the Rossi company S.p.A. accepts the strategy announced by the company Boniello S.p.A., and if the company Boniello S.p.A. alternates the two strategies starting from the fixed price policy, then the sequence of payoffs of the Rossi S.p.A. company can be represented in the following figure:

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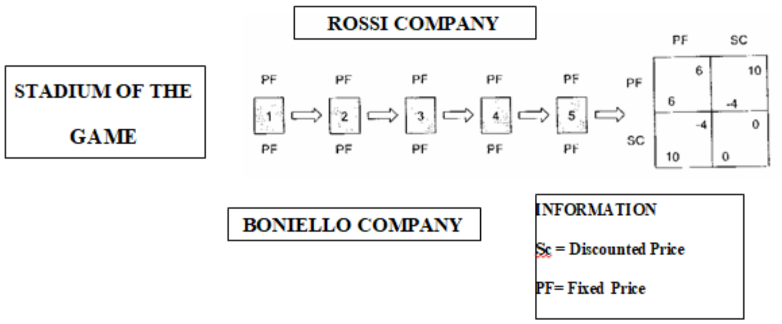
Payoffs obtained by Rossi S.p.A. cooperating with the company Boniello S.p.A.		Payoff of the defection strategy
$6-4\cdot\delta+6\cdot\delta^2-4\cdot\delta^3+6\cdot\delta^4-$	>	10

This means that Rossi SpA can adhere to the strategy announced by Boniello SpA only if the current value of the series of positive and negative alternative payments to the current value (cooperation strategy results), resulting in the current value of the payments obtained by deviating in the first turn (of defection). More briefly, the condition referred to in the previous figure becomes:$6\cdot\frac{1}{1-\delta^2}+(-4)\ \cdot\frac{\delta}{1-\delta^2}>10$. By developing this expression and omitting some passages, we arrive at the condition:$\delta>0{,}20+\sqrt{\frac{11}{25}}=0{,}683$ and so:$i<0{,}158$. Therefore, if the interest rate with which the payments of the company Rossi S.p.A. are discounted is less than 15.8%, then the strategy of always practicing the fixed price is an excellent response to the strategy announced by the Boniello company. This strategy profile, in summary, constitutes a second Nash equilibrium for the game of the company Boniello S.p.A. against Rossi S.p.A. which is in addition to the one identified above. In fact, in addition to the two equilibria discussed, the game in question has many other equilibria, but game theory is unable to indicate which of the various possible equilibria may constitute the predictable outcome of the interaction between the players. As Kreps admits: "in the absence of preliminary negotiations, it is necessary to refer to the existence of conventions, the behaviors learned, a focal point or other equally vague possibilities (...). Alternatively, one could try to explain the particular equilibrium observed as the result of a particular social norm or convention. The theory we have at the moment helps us to understand in general the way in which the repetition of the game allows the formation of many possible focal points or conventional balances. However, for the moment the theory does not have valid formal criteria for choosing a particular equilibrium among the many possible ". In the game of the company Boniello S.p.A. against Rossi S.p.A. it may seem that the equilibrium in which both companies always practice the fixed price policy is the natural outcome of the game. This conclusion appears convincing only if the two firms are in perfectly symmetrical conditions. But, let's assume that the Boniello company is a larger company with more financial resources than the Rossi company. In this case, the Boniello company could announce that it will practice the fixed price and commercial discount policies alternately, and the Rossi company, subject to the condition specified above, may find it appropriate to respond by constantly practicing the fixed price policy. to avoid a trade war that would see it lose. So far we have assumed that a game with the structure of the prisoner's dilemma is played an unspecified number of times. If we change this hypothesis and if instead we assume that the game has a well-defined time horizon, the conclusions reached previously must be radically changed. Let's resume the game that sees the company Boniello S.p.A. on the pitch. against Rossi S.p.A. and suppose the game is played for six rounds. Consider the conjecture that the two firms cooperated in the first five rounds and are about to play the sixth and final repetition of the game. This supposition is not logically defensible. Let's see graphically, to ensure a better understanding, what is highlighted above:



Looking at the graph we notice that while in the first five moves the firms have adopted a cooperative behavior, in the last game turn they know that the possibility of generating with the cooperative behavior a flow of payments whose expected value is higher is no longer possible. to that achievable by adopting the most immediately advantageous move. Consequently, in the last iteration, the dominant strategy for players is that of commercial discounts. If so, when companies reach the fifth stage they know with certainty that the non-cooperative strategy will prevail in the last stage. Therefore, any announcement by companies to adopt a cooperation strategy in the fifth stage has no logical justification and no credibility. But, if in the fifth stage the firms choose the trade discount move, in the previous stage the players are able to anticipate that the cooperative strategy has no chance of being implemented later. Going back from stage to stage, we come to the conclusion, logically unexceptionable, that the only possible equilibrium for the game in question is the one in which companies choose non-cooperative behavior from the very beginning. It is necessary to underline that these conditions allow to reach this conclusion: it is excluded that the players can reach binding agreements before starting to play and also the possibility that the players can modify, stage by stage, their expectations about the behavior of the other firm.

3. Conclusion

The paper was conceived with the intent to highlight the importance of the prisoner's dilemma and its applications. The prisoner's dilemma, in this article we have applied it to three very different events: the writing of the degree thesis, its application in the tobacco manufactures and the role of pricing policies and the prisoner's dilemma. The aim of the paer was to raise awareness of the multidisciplinary nature of the application of the prisoner's dilemma.

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