ISSN: 2456-7760

OPTIMAL NUMBER OF ASSETS FOR REDUCTION OF MARKET RISK THROUGH DIVERSIFICATION

Larissa Adamiec Ph.D.;* Benedictine University

Deborah Cernauskas Ph.D.; Benedictine University

Russell Rhoads; Loyola

Abstract

Market risk can be easily reduced through diversification due to the covariance of the assets. This reduces the overall risk, while maintaining returns, thus achieving the rationale investor's dual goal of maximum returns and minimal risk. Determining the number of assets required to reduce the risk has been long debated. Adding more assets to the portfolio reduces firm specific risk. The downside of adding additional assets is the increased associated trading costs. As a result, investors will want a limit of how many assets to include in their portfolio to gain the optimal level of reduced risk while simultaneously reducing excess trading costs. Most industry professionals estimate a number of assets ranging from 20-30 in a portfolio to reduce the market risk. We seek to test this theory by simulating various portfolios from the S&P500. From the S&P500, 100 individual random portfolios holding 10, 25, 50, 100 and 150 stocks were generated from the years from 1997 - 2016. Each of the portfolios generated returns and standard deviation of returns of the portfolio. These returns and risk metrics were then compared with the overall S&P500. We found the optimal portfolio had 25 stock positions.

Keywords: Diversification, Risk Management, Optimization, Systematic Risk, Unsystematic Risk

Introduction:

Understanding the expected return of a portfolio is rather straightforward as the expected return is a weighted average of the returns of the different assets within the portfolio. Calculating the portfolio variance is more complicated as the variance includes the interaction of the various assets. The interaction of the assets in terms of returns allows for the benefits of diversification. Assets can move either together, opposite of each other or have no relationship at all. Assets which move in opposite directions or assets which have a negative correlation provide natural hedging diversification.

The expected variance of a portfolio of assets is given in equation 1.1. This represents the general formula for variance of a portfolio with n corresponding assets (Bodie, Kane, Marcus 2018).

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(r_i, r_j) \qquad (1.1)$$

ISSN: 2456-7760

The equation 1.1 states that σ_p^2 represents the variance of the portfolio and $w_i w_j Cov(r_i, r_j)$ represents the interaction of each pair of assets within the portfolio.

First, we need to assume the total risk of either a portfolio or asset comes from two sources that is the assets' sensitivity to the changes in the overall market and the assets' specific risk. Total risk can be written as the sum of systematic and unsystematic or firm specific variance. Firm specific variance can be diversified away due to the covariance matrix or the interaction of the various assets within the portfolio. To understand how this works, let us separate equation 1.1 into its two components of systematic and unsystematic risk. Consider a portfolio with equal weights as seen in equation 1.2.

$$w_i = {}^{1}\!/_{n} \qquad (1.2)$$

Since each weight is the same, we can say $w_i = {}^{1}\!/_{n}$ and $w_j = {}^{1}\!/_{n}$ are the same. Equation 1.3 allows us to rewrite the covariance of an asset with itself. We can now transform equation 1.1 to reflect the equally weighted assets which is seen in equation 1.4.

$$Cov(r_i, r_j) = \sigma_i^2 \qquad (1.3)$$

$$\sigma_p^2 = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{n}\sigma_i^2 + \sum_{j=1}^{n}\sum_{i=1}^{n}\frac{1}{n^2}Cov(r_i, r_j) \qquad (1.4)$$

Equation 1.4 allows us to better evaluate the two different sources of risk much more objectively. The first part of the equation is the individual assets specific risks. The second part of the equation is the portfolios sensitivity to the overall market and the interaction of said assets within the portfolio (Amihud, Kamin, Ronen 1983). Separating the two sources of risks allows us to better understand the implications of diversification on the overall portfolio (Markowitz 1991).

Let us define the average variance of the underlying assets of the portfolio as described in equation 1.5. Additionally, let us define the average covariance of the underlying assets of the portfolio as described in equation 1.6. A positive covariance indicates the portfolio's assets will move similarly thus not enjoying the benefits of diversification. On the other hand, a negative covariance indicates the portfolio's assets will move differently thus enhancing the portfolio by reducing risk (Levy, Samat 1970). By defining the average variance and average covariance we can rewrite the total portfolio variance as a combination of the average variance and average covariance as defined in equation 1.7.

$$\overline{\sigma^2} = \frac{1}{n}\sum_{i=1}^{n}\sigma_i^2 \qquad (1.5)$$

$$\overline{Cov} = \frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{n}Cov(r_i, r_j) \qquad (1.6)$$

$$\sigma_p^2 = \frac{1}{n}\overline{\sigma^2} + \frac{n-1}{n}\overline{Cov} \qquad (1.7)$$

As n increases the individual asset specific risk goes to zero. We can now plainly see that as n increases or the number of assets increase the average variance of the assets will go to zero (Chen, Keowan 1981). As the firm specific risk goes to zero, the total risk which is left is the market risk or the portfolio's sensitivity to the overall market. Again, this model assumes there are only two sources of risk to the portfolio: risk from the individual assets themselves and the sensitivity of the risk to the overall market by the portfolio (Saunders, Cornett, McGraw 2006). This graph can be seen in Figure 1: Total Risk to Number of Securities.

Figure 1: Total Risk to Number of Securities

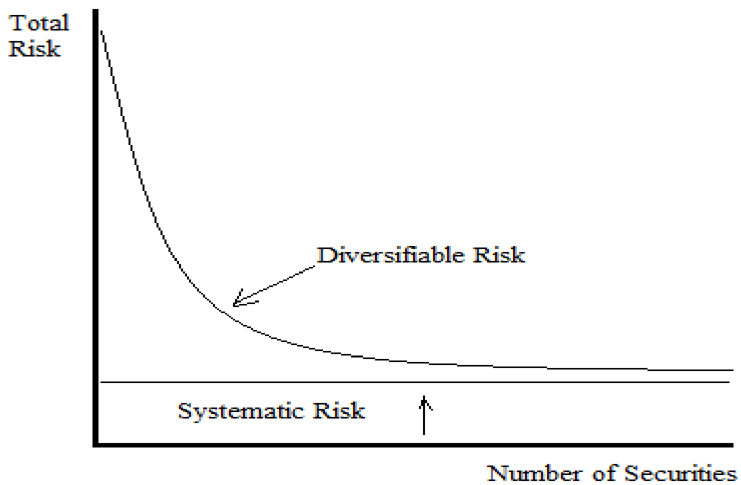

Figure 1 demonstrates as the number securities increase the total risk of the portfolio declines. The diversifiable risk approaches zero as the number of securities in the portfolio increases as depicted in equation 1.4. As this value approaches zero, the total portfolio which is left is the systematic risk or the market risk. The systematic risk then becomes the limit for the overall risk of the portfolio.

The risk of the individual assets or the firm specific risk then can be diversified away. The remaining risk is considered to be systematic risk or market risk. This is the risk associated with portfolio with regards to the market. This risk is consistent with a single factor model, which is often utilized as the Capital Asset Pricing Model (CAPM) (Merton 1973), which is demonstrated in equation 1.8.

$$E[r_i] = r_f + \beta_{i,m}(E[r_m] - r_f) \qquad (1.8)$$

The expected return of the security or portfolio $(i)$is the risk-free rate plus the excess return of the market $(E[r_m] - r_f)$multiplied by the sensitivity $\beta_{i,m}$ of the portfolio. The CAPM says with proper diversification, the only risk to the portfolio is the sensitivity of the portfolio to the market, or simply put market risk. The CAPM is widely accepted by academics as well as practitioners to determine the expected return of either a single asset or a portfolio of assets (Bartholdy, Peare 2005). This one risk factor model is then accepted as an appropriate way to measure returns as determined by the risk of the portfolio to the market.

What is the right number of Assets?

Determining the right number of assets has been a long studied problem. Sharpe developed a brilliant streamlined approach to looking at the relationship between return and risk via the Sharpe ratio (Sharpe 1994). This ratio is the excess risk of the portfolio divided by the variance of the overall portfolio which can be seen in equation 1.9. The Sharpe ratio explicitly evaluates the level of return for each unit of risk. The higher the level of return for a given unit of risk, the higher the level of the Sharpe ratio (Zakamouline, Koekebakker 2009). The ratio now allows us to evaluate a portfolio based on performance of the asset relative to its associated risk.

$$Sharpe = {}^{E[r_p] - r_f}\!\big/_{\sigma_p^2} \qquad (1.9)$$

The Sharpe ratio mirrors the Capital Asset Pricing Model as depicted in equation 1.8. Again, this model evaluates the performance of the portfolio with relation to one single factor, the market. The market risk as demonstrated in both the Capital Asset Pricing Model and the Sharpe Ratio drives the return of the overall portfolio as well as the risk of the portfolio. Investors are therefore properly compensated for the appropriate level of risk of the portfolio as defined by the beta or sensitivity of the portfolio to the overall market (Modigliani, Modigliani 1997).

Eliminating market risk or systematic risk isn't possible utilizing a single factor model of the market. However, eliminating firm's specific or unsystematic risk is with a threshold of n assets. An investor cares to decrease the number of holdings in their portfolio as a way of reducing trading costs or portfolio drag (Elton, Gruber 1977). Utilizing a portfolio of randomly chosen stocks, to have a well-diversified portfolio needs between 30 stocks for a borrowing investor and 40 stocks for a lending investor (Statman 1987).

A challenge of finding the right number of assets is to balance the reduced risk by the cost of adding and subtracting or trading of assets within the portfolio. Although the investor wants to reduce risk to the absolute minimum, the trader does not want to sacrifice returns (Bodie, Rosansky 1980). Trading costs create a drag on the overall portfolio compensation thus reducing the returns for the individual investor or portfolio manager (Odean 1999) and (Barber 2008).

Finding the Optimal Number of Assets

We look to extend this field by applying a more robust simulation on various portfolios. We were able to generate portfolios of five different sizes: 10, 25, 50, 100 and 150. From these different portfolios were able to simulate the returns to determine the empirical returns and variance of each of the portfolios.

To start, the individual total stock return performance for each of the S&P500 from 1997 to 2016 was evaluated. The returns were annual for the years. The average annual return of the individual securities is found in Table 1: Average Annual Returns of Individual Securities in S&P500.

Table 1: Average Annual Returns of Individual Securities in S&P500

Year	Avg. Return	Year	Avg. Return
1997	27%	2007	3%
1998	14%	2008	-38%
1999	13%	2009	44%
2000	10%	2010	21%
2001	1%	2011	1%
2002	-18%	2012	17%
2003	42%	2013	37%
2004	17%	2014	15%
2005	8%	2015	-2%
2006	16%	2016	14%

The average return of for the entire time period was 12%. The highest annual return was in 2003 with an average return of 42%. The lowest annual return was in 2008 with an average return of -38%. A random number of assets were generated for each of the years. The returns of each of the portfolios were calculated. The excess returns over the average return for the entire S&P500 for the individual years were generated as well. These returns are found in Table 2: Average Annual Returns of Simulated Portfolios of the S&P500.

Table 2: Average Annual Returns of Simulated Portfolios of the S&P500

Year	Securities	Avg. Return	Excess	Year	Securities	Avg. Return	Excess
1997	207	33%	6%	2007	224	5%	2%
1998	145	29%	15%	2008	243	-37%	1%
1999	158	21%	8%	2009	292	26%	-17%
2000	313	-9%	-19%	2010	282	15%	-6%
2001	340	-12%	-13%	2011	239	2%	1%
2002	308	-22%	-5%	2012	234	16%	-1%
2003	274	29%	-13%	2013	260	32%	-4%
2004	288	11%	-6%	2014	259	14%	-1%
2005	255	5%	-3%	2015	230	1%	3%
2006	246	16%	0%	2016	262	12%	-2%

Of the 20 years, 8 of the years produced an excess return over the S&P500 index. The other 12 years resulted in the S&P500 outperforming the simulated portfolios. The number of assets which utilized in the portfolio ranged from 145 to 340. The average excess return of the 20 portfolios is -2.70%. The standard deviation of excess returns is 8.02% and the variance of the excess returns is 0.64%. The standard deviation indicates the excess returns have a large swing in excess returns. The descriptive statistics of the annual S&P500 returns, the annual simulated portfolio returns and the annual simulated excess portfolio returns over the S&P500 are depicted in Table 3: Descriptive Statistics of Annualized Portfolios.

Table 3: Descriptive Statistics of Annualized Portfolios

	S&P500	Simulated	Excess
Average	12.09%	9.39%	-2.70%
Standard Dev.	18.47%	18.04%	8.02%
Variance	3.41%	3.25%	0.64%
Skew	-0.73	-0.93	-0.21
Kurtosis	1.79	0.73	0.25
Minimum	-38%	-37.00%	-18.85%
Maximum	44%	33.36%	14.53%

When comparing the simulated portfolios with significantly fewer assets than the simulated portfolios underperformed the S&P500. Taking a deeper look at the return to risk ratio the S&P500 generated a ratio of 0.65 compared with the simulated portfolios of 0.52. The lower return to risk ratio indicates that the simulated portfolios not only underperformed but also generated a lower return compared with unit of risk (Modigliani, Modigliani 1997). These portfolios used significantly higher number of assets compared with the random portfolios tested in this paper.

Results

From the S&P500, 100 individual random portfolios holding 10, 25, 50, 100 and 150 stocks were generated. Each of the portfolios generated returns and standard deviation of returns of the portfolio. These returns and risk metrics were then compared with the overall S&P500.

Table 4: Average Returns of Simulated Portfolios, demonstrates the average returns of the 100 simulated portfolios. The returns of the each of the different portfolios are very similar across each year. The highest average return is not consistent within a certain number of stock portfolio.

Table 4: Average Returns of Simulated Portfolios

	10 Stock	25 Stock	50 Stock	100 Stock	150 Stock		10 Stock	25 Stock	50 Stock	100 Stock	150 Stock
1997	26.99%	27.59%	27.47%	27.41%	27.24%	2007	2.04%	3.09%	4.01%	2.91%	2.99%
1998	14.06%	13.35%	14.25%	13.84%	13.79%	2008	-39.70%	-38.91%	-38.84%	-38.87%	-38.67%
1999	11.66%	11.70%	13.74%	13.12%	13.23%	2009	44.80%	44.06%	43.64%	43.47%	43.93%
2000	9.06%	9.56%	9.97%	9.18%	9.34%	2010	20.96%	20.82%	21.22%	21.00%	21.18%
2001	1.68%	1.16%	0.57%	0.48%	0.57%	2011	0.94%	0.80%	0.81%	1.14%	0.94%
2002	-16.48%	-16.69%	-17.67%	-17.40%	-17.55%	2012	15.99%	15.86%	16.62%	16.52%	16.70%
2003	41.06%	41.38%	41.26%	41.83%	41.85%	2013	37.82%	36.87%	37.42%	36.65%	36.77%
2004	17.66%	17.36%	16.35%	16.99%	16.74%	2014	14.70%	14.82%	15.28%	14.90%	14.93%
2005	8.84%	8.67%	8.50%	8.35%	8.40%	2015	-2.16%	-1.44%	-2.14%	-1.79%	-1.81%
2006	15.33%	15.72%	16.32%	15.95%	15.97%	2016	13.85%	13.25%	13.53%	13.65%	13.71%

ISSN: 2456-7760

Table 5: Standard Deviations of Simulated Portfolios demonstrates the standard deviations of the 100 simulated portfolios for each of the 5 different categories: 10 stock, 25 stock, 50 stock, 100 stock and 150 stock. There is a significant reduction of the standard deviation from the 10 stock portfolios to the 25 stock portfolios. As the number of securities increase the standard deviation decreases for the simulated portfolios.

Table 5: Standard Deviations of Simulated Portfolios

	10 Stock	25 Stock	50 Stock	100 Stock	150 Stock		10 Stock	25 Stock	50 Stock	100 Stock	150 Stock
1997	10.82%	6.66%	4.36%	3.28%	2.59%	2007	11.09%	6.88%	4.58%	2.45%	2.07%
1998	12.06%	8.37%	5.41%	3.70%	3.06%	2008	7.80%	4.66%	3.36%	2.13%	1.55%
1999	14.97%	9.48%	7.77%	5.10%	3.83%	2009	18.29%	10.55%	6.75%	4.64%	3.05%
2000	15.15%	8.91%	5.69%	4.01%	3.06%	2010	8.20%	4.70%	3.88%	2.10%	1.59%
2001	10.41%	6.73%	4.80%	3.33%	2.73%	2011	6.59%	5.03%	3.26%	2.35%	1.84%
2002	8.62%	5.46%	3.58%	2.65%	1.99%	2012	7.13%	4.73%	3.26%	2.39%	1.87%
2003	13.53%	9.21%	5.96%	3.62%	2.73%	2013	10.87%	6.15%	4.66%	2.79%	2.04%
2004	9.90%	5.75%	3.66%	2.55%	1.92%	2014	5.87%	3.90%	3.09%	1.92%	1.53%
2005	7.98%	5.19%	3.83%	2.42%	1.69%	2015	8.28%	5.36%	3.49%	2.34%	1.51%
2006	6.93%	4.08%	3.11%	1.98%	1.59%	2016	8.29%	4.35%	3.01%	2.18%	1.55%

Table 6: Variances of Simulated Portfolios, demonstrates the variances of the 100 simulated portfolios for each of the 5 different categories: 10 stock, 25 stock, 50 stock, 100 stock and 150 stock. There is a significant reduction of the variance from the 10 stock portfolio to the 25 stock portfolio. As the number of securities increase the variance decreases for the simulated portfolios.

Table 6: Variances of Simulated Portfolios

	10 Stock	25 Stock	50 Stock	100 Stock	150 Stock		10 Stock	25 Stock	50 Stock	100 Stock	150 Stock
1997	1.17%	0.44%	0.19%	0.11%	0.07%	2007	1.23%	0.47%	0.21%	0.06%	0.04%
1998	1.45%	0.70%	0.29%	0.14%	0.09%	2008	0.61%	0.22%	0.11%	0.05%	0.02%
1999	2.24%	0.90%	0.60%	0.26%	0.15%	2009	3.35%	1.11%	0.46%	0.22%	0.09%
2000	2.29%	0.79%	0.32%	0.16%	0.09%	2010	0.67%	0.22%	0.15%	0.04%	0.03%
2001	1.08%	0.45%	0.23%	0.11%	0.07%	2011	0.43%	0.25%	0.11%	0.05%	0.03%
2002	0.74%	0.30%	0.13%	0.07%	0.04%	2012	0.51%	0.22%	0.11%	0.06%	0.03%
2003	1.83%	0.85%	0.36%	0.13%	0.07%	2013	1.18%	0.38%	0.22%	0.08%	0.04%
2004	0.98%	0.33%	0.13%	0.06%	0.04%	2014	0.34%	0.15%	0.10%	0.04%	0.02%
2005	0.64%	0.27%	0.15%	0.06%	0.03%	2015	0.69%	0.29%	0.12%	0.05%	0.02%
2006	0.48%	0.17%	0.10%	0.04%	0.03%	2016	0.69%	0.19%	0.09%	0.05%	0.02%

Table 7: Minimums of Simulated Portfolios, demonstrates the minimums of the 100 simulated portfolios for each of the 5 different categories: 10 stock, 25 stock, 50 stock, 100 stock and 150 stock. The minimum was always found in the 10 stock portfolio.

Table 7: Minimums of Simulated Portfolios

	10 Stock	25 Stock	50 Stock	100 Stock	150 Stock
1997	3.79%	7.49%	16.79%	20.41%	20.88%
1998	-8.92%	-5.48%	-0.20%	3.65%	5.37%
1999	-20.76%	-9.30%	-3.33%	-1.34%	3.39%
2000	-29.81%	-20.07%	-6.63%	0.34%	2.69%
2001	-20.00%	-12.79%	-10.21%	-7.20%	-6.28%
2002	-34.97%	-26.47%	-26.94%	-23.31%	-21.83%
2003	15.97%	26.33%	27.00%	33.06%	35.12%
2004	-3.09%	4.21%	8.97%	9.86%	12.95%
2005	-8.04%	-2.63%	-0.61%	3.18%	4.06%
2006	-6.18%	5.88%	7.47%	10.45%	12.59%

	10 Stock	25 Stock	50 Stock	100 Stock	150 Stock
2007	-24.56%	-12.15%	-4.97%	-2.26%	-1.70%
2008	-60.09%	-49.42%	-49.00%	-44.92%	-43.74%
2009	7.35%	23.43%	28.58%	32.13%	35.17%
2010	2.43%	8.84%	12.89%	16.25%	17.92%
2011	-12.27%	-9.37%	-9.32%	-4.40%	-2.76%
2012	-1.07%	4.66%	9.09%	9.67%	11.40%
2013	17.37%	19.44%	26.49%	29.16%	31.25%
2014	0.64%	5.76%	7.85%	8.84%	10.77%
2015	-18.88%	-14.04%	-9.72%	-7.35%	-5.03%
2016	-3.99%	2.76%	6.04%	8.74%	9.72%

Table 8: Maximums of Simulated Portfolios, demonstrates the maximums of the 100 simulated portfolios for each of the 5 different categories: 10 stocks, 25 stock, 50 stock, 100 stock and 150 stock. The maximum was always found in the 10 stock portfolios.

Table 8: Maximums of Simulated Portfolios

	10 Stock	25 Stock	50 Stock	100 Stock	150 Stock
1997	49.78%	45.78%	36.78%	34.18%	33.47%
1998	51.00%	36.21%	25.60%	22.39%	21.25%
1999	73.72%	35.47%	34.76%	25.14%	24.00%
2000	57.45%	35.94%	24.03%	17.97%	16.84%
2001	35.29%	25.74%	11.81%	7.44%	6.41%
2002	8.10%	-3.49%	-8.38%	-10.24%	-12.03%
2003	84.15%	67.76%	56.23%	50.92%	48.14%
2004	48.63%	31.73%	25.64%	23.05%	22.54%
2005	25.93%	20.74%	18.88%	13.96%	12.14%
2006	33.68%	24.54%	24.49%	22.09%	19.66%

	10 Stock	25 Stock	50 Stock	100 Stock	150 Stock
2007	27.76%	20.67%	17.74%	7.68%	7.47%
2008	-18.60%	-25.25%	-29.49%	-33.31%	-34.08%
2009	103.57%	72.56%	63.35%	56.98%	52.36%
2010	43.03%	30.53%	30.14%	25.82%	24.78%
2011	19.36%	10.62%	6.58%	5.31%	5.07%
2012	36.41%	28.62%	26.66%	23.70%	23.08%
2013	86.53%	58.57%	49.96%	43.91%	40.82%
2014	31.47%	23.88%	23.34%	21.21%	18.28%
2015	20.67%	11.55%	6.34%	4.22%	1.84%
2016	38.89%	23.98%	23.43%	18.77%	17.18%

Conclusions

The results of the testing are found in Table 9: Average Returns of Random Portfolios.

Table 9: Average Returns of Random Portfolios

Stocks	Average Annual Return	Average Annual Volatility	Perecent Outperform
10	11.95%	21.27%	86%
25	11.95%	19.52%	96%
50	12.12%	19.10%	100%
100	11.97%	18.77%	100%
150	12.01%	18.71%	100%
S&P 500 TR	9.39%	18.04%	N/A

ISSN: 2456-7760

The high average returns are not consistent through the different portfolio sizes of 10, 25, 50, 100 and 150 stock positions. As a result, we can not draw a conclusion. The standard deviation and variance decreases as the number of assets increases. We have demonstrated this in equation 1.4. The simulated portfolios empirically demonstrate the reduction of the overall risk as the number of securities increase. With the minimum and maximum in the 10 stock portfolios, we can see the extremes of the average of the 100 simulated portfolios.

Works Cited

Bodie, Kane. "Marcus. Investments." (1999).

Statman, Meir. "How many stocks make a diversified portfolio?." *Journal of financial and quantitative analysis* 22.3 (1987): 353-363.

Evans, John L., and Stephen H. Archer. "Diversification and the reduction of dispersion: an empirical analysis." *The Journal of Finance* 23.5 (1968): 761-767.

Elton, Edwin J., and Martin J. Gruber. "Risk reduction and portfolio size: An analytical solution." *The Journal of Business*50.4 (1977): 415-437.

Sharpe, William F. "The sharpe ratio." *Journal of portfolio management* 21.1 (1994): 49-58.

Modigliani, Franco, and Modigliani Leah. "Risk-adjusted performance." *Journal of portfolio management* 23.2 (1997): 45.

Elton, Edwin J., and Martin J. Gruber. "Risk reduction and portfolio size: An analytical solution." *The Journal of Business*50.4 (1977): 415-437.

Merton, Robert C. "An intertemporal capital asset pricing model." *Econometrica* 41.5 (1973): 867-887.

Amihud, Yakov, Jacob Y. Kamin, and Joshua Ronen. "‘Managerialism’,‘ownerism’and risk." *Journal of Banking & Finance* 7.2 (1983): 189-196.

Markowitz, Harry M. "Foundations of portfolio theory." *The journal of finance* 46.2 (1991): 469-477.

Levy, Haim, and Marshall Sarnat. "International diversification of investment portfolios." *The American Economic Review*60.4 (1970): 668-675.

CHEN, SON-NAN, and Arthur J. Keown. "Risk decomposition and portfolio diversification when beta is nonstationary: A note." *The Journal of Finance* 36.4 (1981): 941-947.

Saunders, Anthony, Marcia Millon Cornett, and Patricia Anne McGraw. *Financial institutions management: A risk management approach*. New York, NY, USA: McGraw-Hill, 2006.

Bartholdy, Jan, and Paula Peare. "Estimation of expected return: CAPM vs. Fama and French." *International Review of Financial Analysis* 14.4 (2005): 407-427.

Odean, Terrance. "Do investors trade too much?." *American economic review* 89.5 (1999): 1279-1298.

Barber, Brad M., et al. "Just how much do individual investors lose by trading?." *The Review of Financial Studies* 22.2 (2008): 609-632.

Zakamouline, Valeri, and Steen Koekebakker. "Portfolio performance evaluation with generalized Sharpe ratios: Beyond the mean and variance." *Journal of Banking & Finance* 33.7 (2009): 1242-1254.

Bodie, Zvi, and Victor I. Rosansky. "Risk and return in commodity futures." *Financial Analysts Journal* 36.3 (1980): 27-39.