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ANALYSIS OF STOCHASTIC PROCESSES: CASE OF AUTOCORRELATION OF EXCHANGE RATES

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Abstract

The present work consists of explaining how to study stochastic processes and to apply them to different economic and financial series. In this context, we chose the case of exchange rates by comparing the euro to the three major international currencies (US dollar, pound sterling and Japanese yen). Emphasis is placed on the existence or not of the autocorrelation phenomenon of the three series of exchange, over the period 2009-2011, proceeding in a first case, to tests of white noise (Box-Pierece [1970] and Ljung-Box [1978]) to the initial raw series, and in a second case, to the same series calculated as a first difference. The results are very different. The choice of the period is dictated by the context of high volatility in the markets that followed the 2008 financial crisis.

Keywords: Stochastic process, stationary process, white noise, autocorrelation, correlogram, exchange rate

General Introduction :

The study of stochastic processes plays a central role insofar as many theoretical models of time series representations assimilate the values actually observed with stochastic process realizations. These processes have often found their application in the financial and economic field.

In this framework, we analyze some cases of stochastic processes observed in the financial series, such as the evolution of exchange rates. In recent years, we are witnessing excessive volatility in the financial markets. For this reason, we have chosen to study series of euro exchange rates against three major international currencies, namely: the US dollar, the British pound and the Japanese yen.

In this context, we are interested in the study of the behaviors of the different time series by trying to test the stationarity of the stochastic processes and the existence or not of white noises in these series.

Historical data are downloaded from the FININFO database for the period December 2008 to December 2011.

To carry out this study, we chose to proceed by the following steps:

- Theoretical framework of the study and definitions

- Issues

- Methodology and data

- Results: analysis and interpretation

- Conclusion

1- Theoretical framework: Definitions and main types of stochastic processes

What does a stochastic process mean? What are the different types of stochastic processes?

A random variable is said to follow a stochastic process when the changes in value of that variable over time are, at least partly random.

The two terms random and stochastic are synonymous.

To designate the variable, the term random is preferably used. We say a **random variable.**

To designate the process that takes place over time, the term stochastic is preferably used. We say a **stochastic process.**

When changes in the value of the variable occur only at discrete points of time, we speak of **discrete time processes.** When changes can occur at any time, it is said to be a **continuous time process.** We therefore speak of discrete **variable process and continuous variable process**.

A variable that has a stable probability distribution for any translation over time follows a **stationary process** (ie that samoyenne or mathematical expectation is constant and its finite variance constant). On the other hand, if its probability distribution changes, especially its expectation and variance, it is said to follow a **non-stationary process.**

The set of realizations of the variable (discrete or continuous) is called a trajectory during a time interval.

One type of stochastic process is of great importance in finance: it is the **Markov process**. The future values of a variable that follows a Markov process:

- depend on the value taken by the variable at the moment

- depend only on this value and not on previous values.

Markov processes in continuous time and continuous variable are called **diffusion processes.**

The **simple Brownian motion,** also called **the Wiener process,** is a continuous and continuous variable Markov process. His random increments are independent of each other. The increase that occurs during a finite time interval has a normal distribution whose variance increases with the length of the interval.

A **white noise (BB) is a particular stationary stochastic** process which represents a sequence of random variables εt, of zero mean and of constant variance for all the values of t and of zero covariance for all the values of t and t '(with t ≠ t '), i.e. :

$E(\varepsilon t) = 0$

$Var(\varepsilon t) = \sigma^2$

$Cov(\varepsilon t, \varepsilon t') = 0$, for any time

An Xt process is **called a random walk or a random walk** if:

$Xt = Xt\text{-}1 + \varepsilon t$ where εtest a white noise (BB)

We can also write:

$$X_t = \sum_{k=1}^{t} \varepsilon_k$$

so that the expectation of a random walk is equal to 0 and its variance to $t\sigma^2$ (the process is not stationary).

This process is of great interest in finance since most financial time series are assumed to follow a random walk, the subject of this study.

2- Issue

The purpose of this study is to analyze the behavior of time series relating to the exchange rate of certain currencies in relation to the Euro.This is justified by the fact that the European currency is subject to strong fluctuations, which threatens the Euro zone by a total collapse.

For this, this work attempts to answer the following questions:

- To what extent are these exchange rate series equated with stationary stochastic processes?

- Are these series purely random or show trends?

- What is the importance of white noise in all the financial series analyzed?

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3- Methodology and data

3.1. Data

The studied data are obtained from the FININFO database (database distributed by SIX-TELEKURS). These are the daily exchange rate series for the period from 23/12/2008 to 26/12/2011 for the euro against three international currencies: the US dollar, the Sterling Pound and the Japanese Yen. Quotes are those of the London spot market (TPI)

3.2. Methodology

In this section, we analyze for each series, firstly, the descriptive statistics, ie. the mean, the standard deviation, the skewness (asymmetry coefficient) and the kurtosis (flattening coefficient). This study will allow us later to know if the different historical sequences follow a normal law or not (test of normality through the Jarque-Bera test (1980).

Calculation of the JB Statistics:

$$JB = \frac{n-2}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right)$$

JB follows a chi-square law (□²) with two degrees of freedom and the threshold 1-□.

Where S coefficient Skewness, and K, kurtosis coefficient

Test: if JB> □² with two degrees of freedom and with the threshold 1-□., We reject the hypothesis H0 of normality at the threshold □ = 5%.

Next, we test the randomness of these different series, ie, whether these stochastic processes are random or trendy. If the series includes trends, what is the degree of auto correlation?

To answer these questions, we proceed to the Parametric tests of Box-Pierce (1970).

Q statistics of Box-Pierce:

$$Q = n\sum_{k=1}^{h} r_k^2$$

where: h = number of delays, rk = empirical autocorrelation of order k,

n = number of observations

h must be close to 1/5 of the number of observations.

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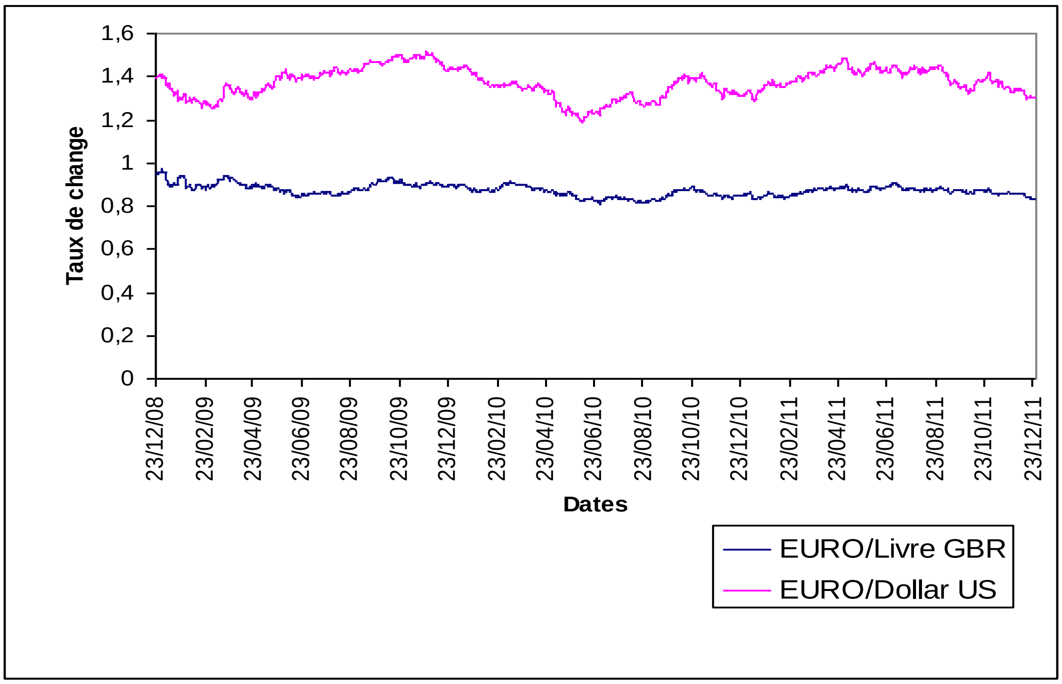
The Q statistic is asymptotically distributed as a □² (chi-square) with h degrees of freedom. We reject the white noise hypothesis, at the threshold □, if the statistic Q is greater than the □² read in the table at the threshold (1-□) and h degrees of freedom.

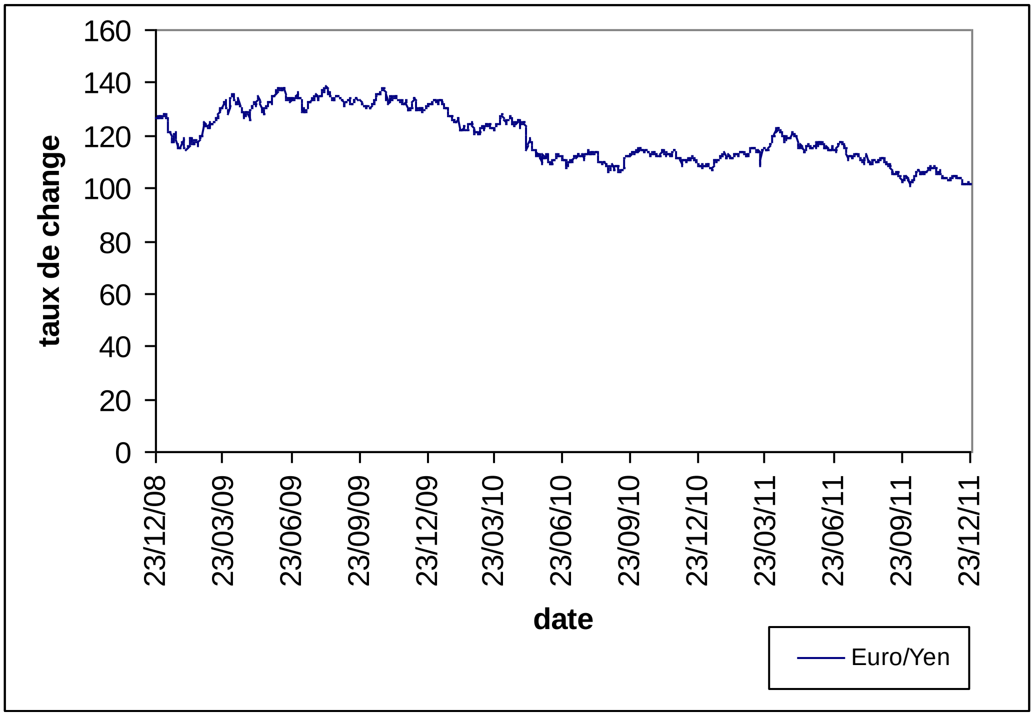
4- Results: analysis and interpretation

4.1. Exchange rate graph of the euro against pound sterling, dollar and yen

Chart1. EURO against Pound Sterling and US Dollar



Graph 2: EURO against Japanese Yen



4.2. Descriptive statistics and normality test

	EUR/LIVRE GBR	EUR/YEN JPN	EUR/DOLLAR USA
Moyenne	0,87308	119,31983	1,37191
Ecart type	0,02716	10,16723	0,06716
Skewness	0,39484	0,23473	-0,25904
Kurtosis	0,37578	-1,23995	-0,52070
JB	330,757907	801,451068	557,732820

We find that the JB statistic of the 3 exchange rate series is greater than the CHI-DEUX value calculated at the alpha threshold = 5% at 2 degrees of freedom which is equal to 5.991.

Therefore, we conclude that there is no normality for the three Euro exchange rate series at only alpha = 5%,

4.3. Auto correlation detection and white noise test

Q statistics of Box-Pierce:

$$Q = n\sum_{k=1}^{h} r_k^2$$

where: h = number of delays, rk = empirical autocorrelation coefficients of order k,

n = number of observations

If xt is a white noise (iid - independent and identically distributed), then Cov (xt, xt-k) = 0 or rk = 0, □k, hence the hypothesis test:

Hypothesis test:

- H0: r1 = r2 = ... = rk = 0

- H1: there is at least one ri with i□ 0 (significantly different from 0)

k ranging from 1 to 80

In Table 1 below, we present the autocorrelation coefficients in initial raw series of order 1 to 80

Table 1: The autocorrelation coefficients in raw series of order 1 to 80

k	r_k (Euro/Livre)	r_k (Euro/Yen)	r_k (Euro/Dollar)
1	0,9834	0,9954	0,9913
2	0,9648	0,9910	0,9828
3	0,9473	0,9862	0,9738
4	0,9307	0,9821	0,9659
5	0,9132	0,9783	0,9581
6	0,8958	0,9753	0,9515
7	0,8782	0,9715	0,9448
8	0,8613	0,9679	0,9374
9	0,8446	0,9643	0,9290
10	0,8281	0,9605	0,9206
11	0,8134	0,9563	0,9118
12	0,8006	0,9520	0,9027
13	0,7869	0,9480	0,8937
14	0,7761	0,9446	0,8857
15	0,7663	0,9411	0,8769
16	0,7569	0,9381	0,8676

k	r_k (Euro/Livre)	r_k (Euro/Yen)	r_k (Euro/Dollar)
17	0,7488	0,9351	0,8589
18	0,7420	0,9319	0,8502
19	0,7338	0,9284	0,8416
20	0,7252	0,9245	0,8324
21	0,7164	0,9203	0,8232
22	0,7073	0,9165	0,8145
23	0,6989	0,9128	0,8052
24	0,6917	0,9089	0,7961
25	0,6829	0,9050	0,7870
30	0,6173	0,8849	0,7404
40	0,5100	0,8578	0,6584
50	0,4567	0,8410	0,5852
60	0,4004	0,8293	0,5249
70	0,3536	0,8149	0,4613
80	0,2869	0,7904	0,3974

The statistics of Q

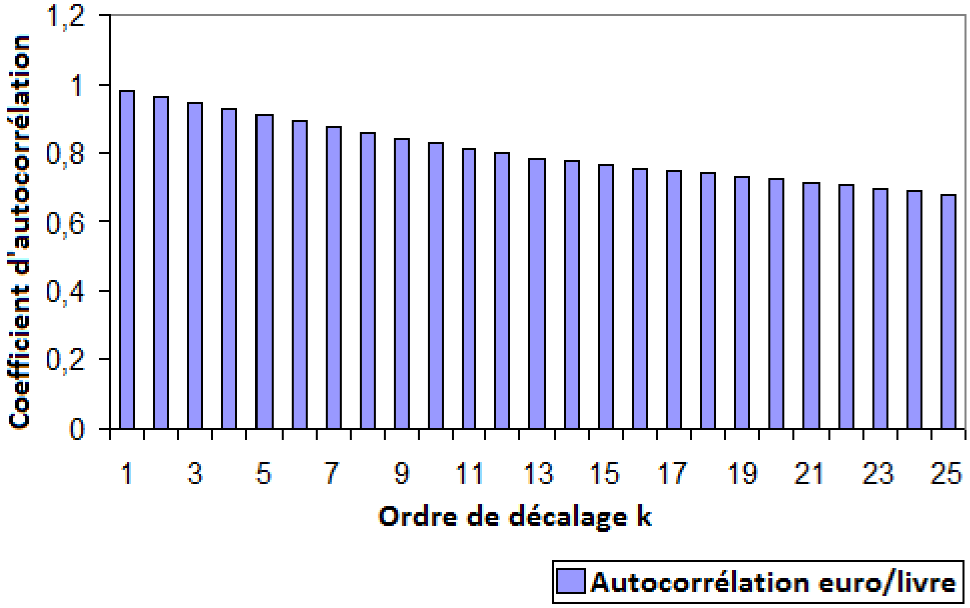
	EUR/LIVRE	EUR/YEN	EUR/DOLLAR
Valeur Q	**18780,68**	**28334,93**	**23256,33**

The value of CHI-DEUX at the alpha threshold = 5% and at h = 80 degrees of freedom is equal to 101.59. Therefore, the value of Q is greater than chi-square for h = 80 degrees of freedom, so we find that the 3 series are autocorrelated and do not show random processes.

It follows that the three series are autocorrelated and do not show random processes at the threshold □ = 5%.

The same results are visual through the graph of the correlogram. The latter graphically represents the autocorrelation coefficients (see graph 3)

Graph3: Correlogram of the exchange rate from euro to pound sterling



Will the results be the same by studying the same series but in first difference?

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4.4. Graph of exchange rates of the euro against pound sterling and the dollar in first differences

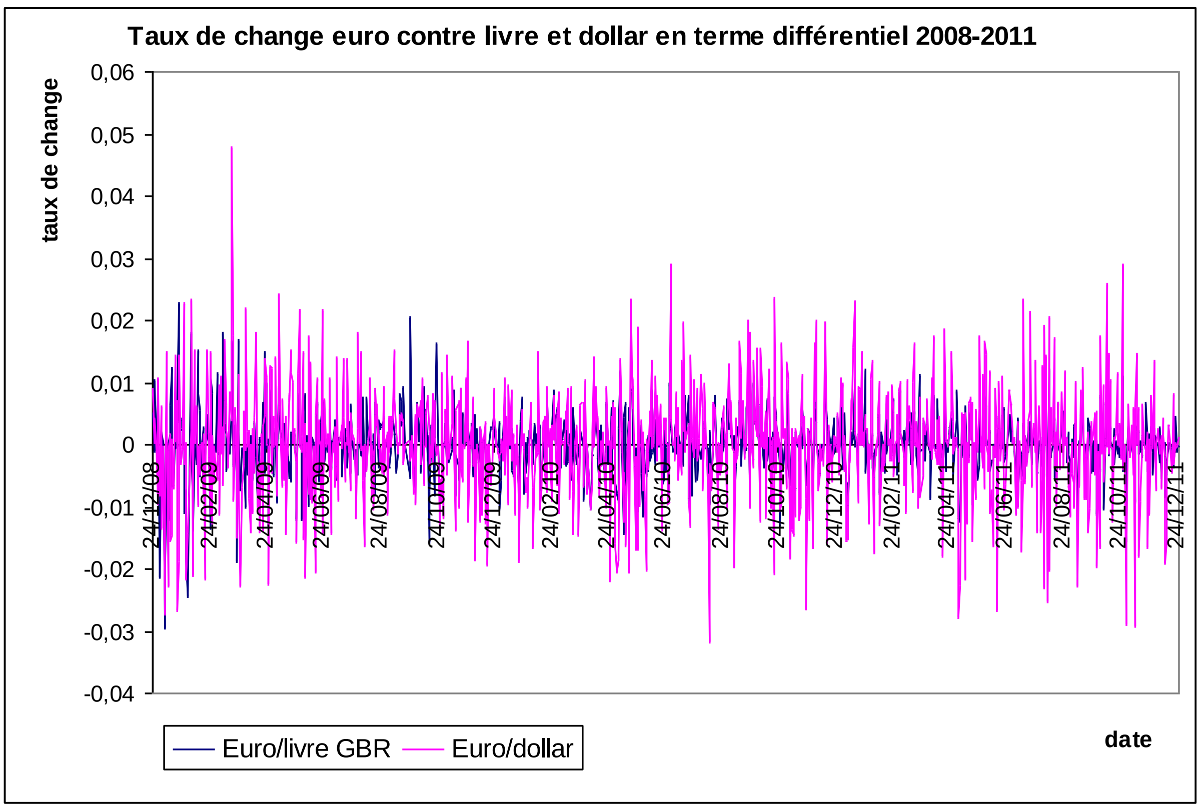
Let's analyze the same series not in their initial value, but in first difference.

Let Δt be the first difference of X between time t and t-1,

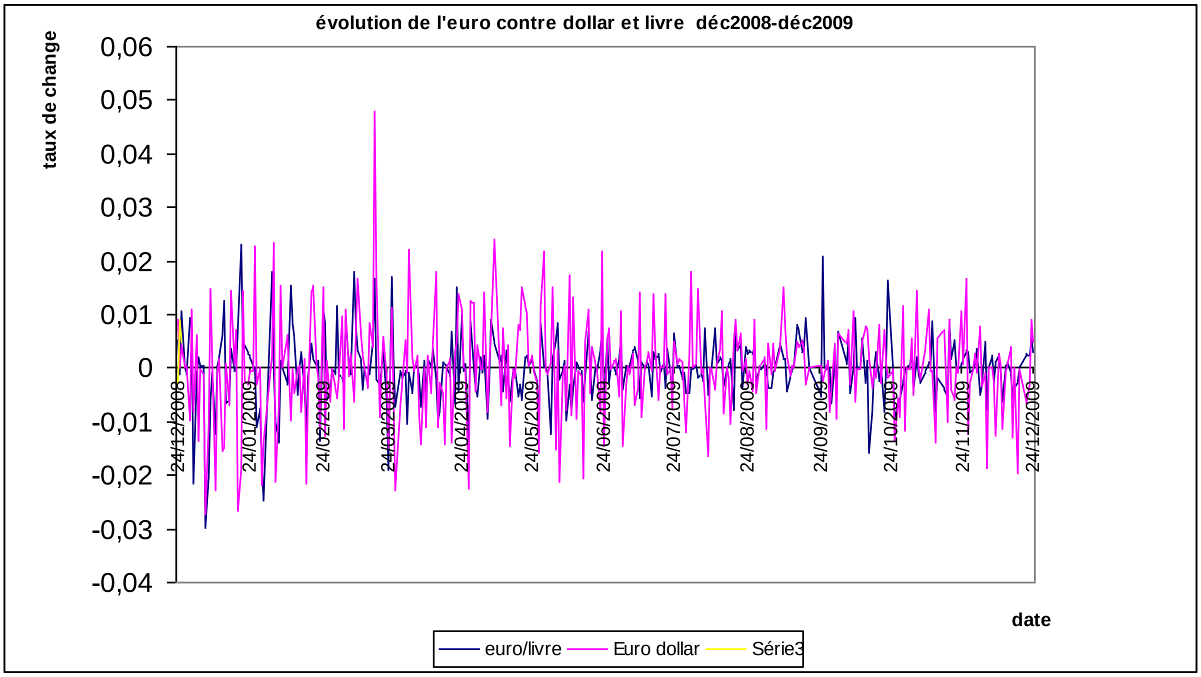
$\Delta t = Xt - Xt-1$

The two charts (4 and 5) of the first difference series are entirely different from the first case when the series of exchange are analyzed in their initial value. This suggests a lack of autocorrelation between different series of exchange.

Graphe4. EURO against British pound and US dollar (first difference): 2008-2011



Graphe5. EURO against Pound Sterling and US Dollar (in first difference): December 2008-December 2009



4.5. Descriptive statistics and normality test for differential series

	EUR/LIVRE GBR	EUR/YEN JPN	EUR/DOLLAR USA
Moyenne	-0,00010	-0,02344	-0,00008
Ecart type	0,00494	0,97728	0,00886
Skewness	-0,31327	-0,45809	-0,05189
Kurtosis	3,71738	3,49790	1,65134
JB	39,91606	47,84026	80,50420

We find that the JB statistic of the 3 exchange rate series is greater than the CHI-DEUX value calculated at the alpha threshold = 5% at 2 degrees of freedom which is equal to 5.991. We also note that the JB value has significantly decreased in differential terms compared to the initial raw series.

Therefore, we conclude that there is no normality for the three Euro exchange rate series at only alpha = 5%,

4.6. Auto correlation analysis and white noise test for differential series

In Table 2 below, we present the autocorrelation coefficients with a first order difference of 1 to 80

Table 2: The autocorrelation coefficients in first order difference from 1 to 80

k	r_k(Euro/Livre)	r_k(Euro/Yen)	r_k(Euro/Dollar)
1	0,05634	-0,02268	-0,00961
2	-0,04053	0,03358	0,02537
3	-0,02773	-0,07207	-0,06317
4	0,02111	-0,03702	-0,00823
5	-0,00757	-0,07346	-0,06240
6	0,00223	0,06867	-0,00271
7	-0,02237	-0,01800	0,04456
8	-0,01954	0,00574	0,05711
9	-0,00862	0,01151	-0,00323
10	-0,06034	0,04887	0,02283
11	-0,05547	-0,00094	0,01496
12	0,01858	-0,02747	-0,00249
13	-0,06688	-0,06362	-0,05524
14	0,00050	0,01632	0,05027
15	0,00540	-0,06353	0,02909
16	-0,03719	0,00093	-0,03312
17	-0,01183	0,04101	0,01015
18	0,04062	0,03242	-0,00946
19	0,00626	0,03465	0,03806
20	-0,00587	0,04184	0,00408
21	-0,01769	-0,02577	-0,01168
22	-0,00747	-0,00338	0,03009
23	-0,01972	0,00073	-0,00945
24	0,03693	-0,00633	-0,01331
25	0,00044	-0,03581	-0,01355
30	-0,01406	-0,01685	0,00125
40	0,00475	0,01754	0,03151
50	0,06581	-0,03277	0,00389
60	-0,01461	-0,00855	0,06534
70	0,01943	-0,00377	-0,03885
80	-0,03520	-0,03717	-0,01012

The statistics of Q

	EUR/LIVRE	EUR/YEN	EUR/DOLLAR
Valeur Q	32,12506	43,50227	34,13681

The value of CHI-DEUX at the alpha threshold = 5% and at h = 80 degrees of freedom is equal to 101.59.

The empirical Q statistic for the 3 euro exchange rate series is less than chi-square for h = 80. So we find that the 3 series are independent and follow a random walk in terms of first differences.

The series are therefore stationary in differential terms.

We deduce that the three series are independent and follow a random walk in terms of first differences at the threshold □ = 5%.

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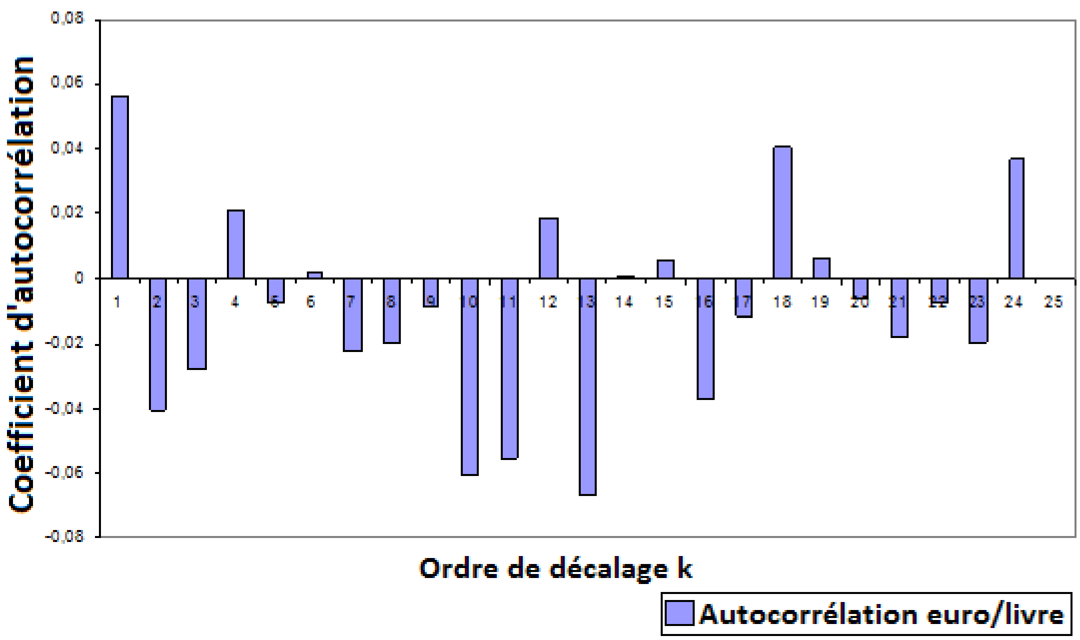
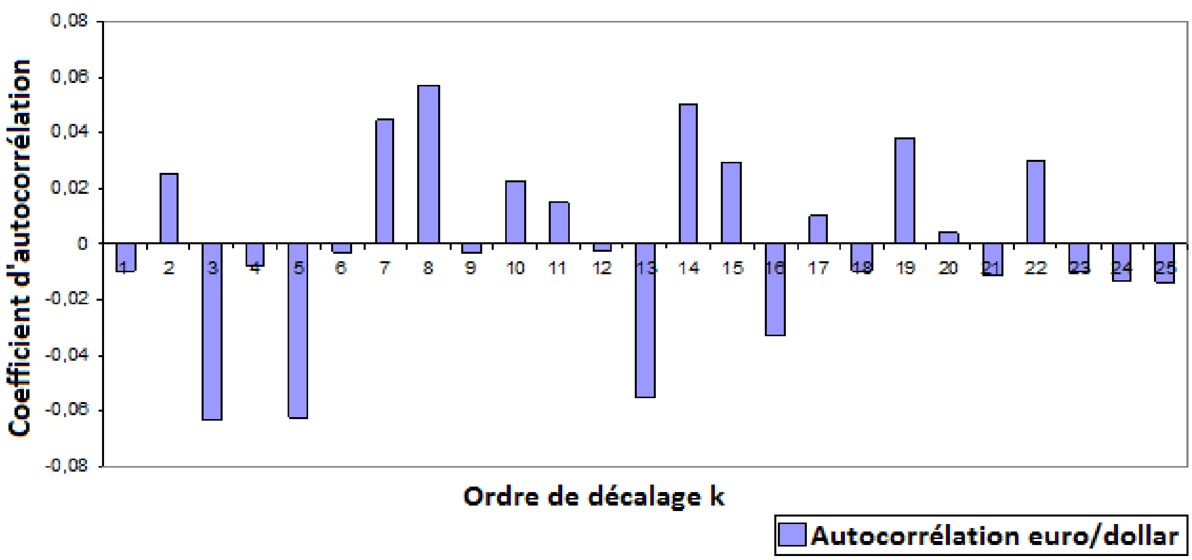
These results are confirmed by the euro correlogram against pound and dollar.

Graph 6: Exchange rate correlation of the euro / pound sterling in differential series



Graph 7: Correlogram of the exchange rate of the euro / dollar in differential series



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5. Overall conclusion

In conclusion, we can conclude that the 3 series of exchange rates of the euro against the dollar, the pound sterling and the Japanese yen are indeed purely random stochastic processes. If the autocorrelation tests make it possible to assert that the different series include trends and are therefore deterministic in their initial values, their transformation into first differences causes the disappearance of this phenomenon and shows that these processes follow a random walk because of white noise they contain. This is consistent with the theory of financial market efficiency.

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Used tools:

FININFO database (SIX-TELEKURS provider)

Excel spreadsheet