
**IDENTIFICATION AND ANALYSIS OF ALTERNATIVE METHODOLOGIES FOR
THE ESTIMATION OF THE EXPECTED VALUE OF AN ASSET'S PORTFOLIO**

Eralda Caushi

University of Lady of Good Council
ALBANIA

ABSTRACT

The aim of this paper is to identify and analyze the strategies that characterize the dynamic asset allocation. In literature we have found 4 strategies: Buy and hold, Constant Mix, Constant-proportion portfolio insurance and option-based portfolio insurance (Perold & Sharpe, 1995). The insurance based ones are not in our focus so we will make an evaluation between the two first strategies. Risk propensity is different for any investor. Persons who are "risk averse" have a low risk propensity; those who are "risk takers" have a high risk propensity. The prices follow a stochastic process so the expected value will also depend on some random measure, which is the inverse of normal (0.1). Calculations are made for each level of significance observed. Comparison is therefore made between the expected values of the portfolio from year to year throughout the investment period, for both strategies and each observed significance level and risk propensity. The question at the base of the study is which strategy of dynamic allocation is appropriate in case of an asset's market which has a well-defined upward trend, and an accentuated and unpredictable volatility.

Keywords: VaR, risk, return, simulator

INTRODUCTION

Asset allocation is the distribution of the funds available between the various investment activities. The investor after choosing how to invest his own portfolio is in the phase of the dynamic asset allocation process. At this stage, it must periodically monitor the performance of the portfolio and, according to its own risk profile, decide to make any changes in the composition of the portfolio. There are four possibilities among which the investor can choose:

- Do not do anything (Strategy is defined as Buy & Hold)
- Redeem the asset class weights, returning percentage shares to the starting values (The strategy is defined as Constant Mix)
- Constant-proportion portfolio insurance (CPPI) strategies are constant-proportion strategies with multiplier greater than one
- Option-based portfolio insurance (OBPI) strategies begin by specifying an investment horizon and a desire floor at that horizon.

This process consists essentially in identifying the expected value of a portfolio compound of assets during the investment period. During this period, the various asset classes that will make up the portfolio considered change in value according to the financial market trend. That's the reason why the weights of the asset class change from year to year, in line with the trend of the financial markets. At this point, you just have to choose the dynamic allocation to fit your portfolio. A very important variable here to take into account is the value-at-risk (VaR), defined as the maximum potential loss of a particular portfolio, where it can be incurred with a certain level of confidence for a certain future time. In this paper we will also have a brief of the main concepts of the financial market, assets, financial instruments and the theory of the portfolio. The thesis aims to individualize and analyze the two strategies that characterize dynamic asset allocation and make a comparison between these two strategies. Comparisons of the strategies are made in the following three directions. The prices follow a stochastic process so the expected value will also depend on some random measure, the inverse of normal (0.1). Calculations are made for each level of significance observed. Comparison is therefore made between the expected values of the portfolio from year to year throughout the investment period, both for the Buy & Hold strategy and the Constant Mix strategy for each level of significance observed. Each investor has a risk propensity different from that of other investors. Investor risk profiles are defined according to the percentages that the three asset classes (short term bonds and stocks) report in the portfolio considered. We can then confirm that the comparison is based on strategy, level of significance and risk aversion. The question at the base of the research is what strategy of dynamic allocation is appropriate if the asset market has a well-defined upward trend, and what if the market is very volatile and unpredictable. The research is substantially base in the below work phases:

- I. Building an excel application for calculating the expected value of the portfolio considered under the Buy and Hold strategy.
- II. Building an Excel Application for Calculation of Expected Portfolio Values Considered Constant Mix Strategy.
- III. Comparison between the two strategies each year for each level of confidence, and for each risk profile.
- IV. Conclusion in the case of volatile and unforeseeable markets.

LITERATURE REVIEW

Modern portfolio theory (MPT), or mean-variance analysis, is a mathematical framework for assembling a portfolio of assets in order to have a maximized expected return for a given level of risk, defined as variance. So, the MPT methodology is based on the average/variance

optimizer. The optimization function is based on Markowitz's portfolio theory and on the efficient frontier concept. A combination of assets, i.e. a portfolio, is referred to as "efficient" if it has the best possible expected level of return for its level of risk (which is represented by the standard deviation of the portfolio's return). Here, every possible combination of risky assets can be plotted in risk–expected return space, and the collection of all such possible portfolios defines a region in this space. In the absence of the opportunity to hold a risk-free asset, this region is the opportunity set (the feasible set). The positively sloped (upward-sloped) top boundary of this region is a portion of a parabola and is called the "efficient frontier." If a risk-free asset is also available, the opportunity set is larger, and its upper boundary, the efficient frontier, is a straight line segment emanating from the vertical axis at the value of the risk-free asset's return and tangent to the risky-assets-only opportunity set. All portfolios between the risk-free asset and the tangency portfolio are portfolios composed of risk-free assets and the tangency portfolio, while all portfolios on the linear frontier above and to the right of the tangency portfolio are generated by borrowing at the risk-free rate and investing the proceeds into the tangency portfolio (Harvey, May 2017)

Its key insight is that an asset's risk and return should not be assessed by itself, but by how it contributes to a portfolio's overall risk and return. The model allows the user to manually define assets. The system retrieves the list of the various asset classes and the user must define which ones to use and the percentage within the portfolio. The system calculates the efficient frontier and suggests to the user an optimal portfolio, depending on its risk profile. Economist Harry Markowitz introduced MPT (Markowitz, 1952), for which he was later awarded a Nobel Prize in economics.

METHODOLOGY

Prices and Return

The historical time series of the Price variable has an evolution that can be attributed to a random walk process. This assumption is consistent with the market efficiency hypothesis: Information is completely embedded in the price, with some constraints on the independence (or incorrectness) between price variations. The equation characterizing a random walk process (with drift) is:

$$p_t = \mu + p_{t-1} + u_t$$

Where $E(u_t | p_{t-1}, p_{t-2}, \dots) = 0$, μ is the deterministic constant, p_{t-1} : price found on the previous date, u_t is the random term. These series have the covariance moments that depend on time (t), so they are not stationary. Simple returns follow a log-normal distribution:

$$r_t = \frac{P_t}{P_{t-1}} - 1$$

In order to be in accordance with the normal distribution hypothesis, reference is made to logarithmic returns:

$$\tilde{r}_t = \ln(P_t) - \ln(P_{t-1})$$

Risk and Return

A portfolio is a set of financial assets held by an investor. It can therefore consist of equities and/or bonds, derivatives or other asset classes. Portfolio theory deals with optimal investment decisions by risk-averse investors. They have the objective to maximize the expected return on their portfolio for a given level of risk they deem acceptable, or to minimize the risk they are exposed to for a given target return. The return of a portfolio is given by the formula:

$$\mu = \sum_{t=1}^T w_t \cdot r_t$$

w_t is the weight of the r_t at the time t. In matrix computation you can write: $\mu = W^T R$

where W^T is the weight matrix, and R is the matrix of returns defined as: $R = (r_1 \dots r_t)^T$. The portfolio variance is the weighted sum of the variance of each of the assets to which the covariance is added.

$$\sigma = W^T \Sigma W$$

W^T is the weighing vector and Σ is the variance-covariance matrix. The investor's behavior follows the mean-variance criterion according to which the investor prefers a higher average return than a lower average return for a certain level of risk. Or, it prefers a lower risk than a higher risk to a given average return. Between A and B portfolios that have a different composition, the investor chooses portfolio A to B if:

$$\left. \begin{aligned} E(r_A) &\geq E(r_B) \\ \sigma_A &\leq \sigma_B \end{aligned} \right\} \text{CRITERION MEAN-VARIANCE}$$

Therefore, the average variance policy does not determine the optimal portfolio, but a set of efficient portfolios that make up the efficient frontier. Each portfolio is exposed to risks that can be eliminated or not. The risk of a portfolio that can be potentially eliminated by diversifying the portfolio is called specific risk. This risk stems from the fact that many of the dangers

surrounding a single business are particular to this. This risk is known as systemic risk or market risk. Systematic risk derives from the fact that there are problems and dangers affecting the entire economy, posing a threat to all activities. Usually a portfolio of different activities is considered. Choosing to consider a portfolio and not just one title is a way to reduce variability and thus the risk of the portfolio. This is a consequence of diversifying a portfolio of assets. The risk of a well-diversified portfolio depends on the systematic risk of the securities included in the portfolio. Then the specific risk diversifying, it is eliminated. In order to know the contribution of a single asset to the risk of a well-diversified portfolio, it is necessary to measure its systematic risk. This shows us how much the stock is sensitive to market movements. This sensitivity to the performance of an investment in market movements is usually called beta (β). The beta coefficient is intended as a measure of the ratio between the degrees of variability of a business's performance compared to changes in the financial market all together. Expressed by the formula:

$$\beta_j = \frac{\text{cov}(R_m, R_j)}{\text{var}(R_m)} = \frac{\rho_{j,m} \sigma_j \sigma_m}{\sigma_m^2} = \rho_{j,m} \frac{\sigma_j}{\sigma_m}$$

$\text{cov}(R_m, R_j)$ is the covariance existing between the performance expressed by Market portfolio R_m and R_j activity. $\text{var}(R_m)$ is the variance of the market portfolio returns and $\rho_{j,m}$ is the linear correlation coefficient between the j activity and the return on the market portfolio. σ is the standard deviation of returns.

Value at Risk

Value at Risk (VaR) is defined as the maximum potential loss of a particular portfolio, with a certain level of confidence for a certain future time. Alternatively, the VaR is defined as the maximum amount you expect to lose with a given level of confidence, measured in relationship to the wealth you expect to get at the end of the period. The value at risk was introduced for the first time by JPMorgan. The concept of VaR, as deduced from the definition, consists in the choice of two parameters: the period, which can be daily, weekly, monthly, quarterly or otherwise; and the confidence level, which may be 90%, 95%, 99%, or other probability of choice. Let's explain how we can derive the formula that determines the Value at Risk: We know that the wealth and the average return on a diversified portfolio are linked by the formula:

$W_t = (1 + R_t)W_{t-1}$, W_t is the wealth, and R_t is the return, at t time. In our model, we use compound returns (a k periods):

$$1 + R_t(k) = \frac{P_t}{P_{t-k}} = \prod_{j=0}^{k-1} (1 + R_{t-j}).$$

If we assume that $(1 + R_t)$ are independent and distributed identically, during the period t, then the process is a random walk geometric process. We also assume that $(1 + R_t)$ has a lognormal distribution so we have:

$$r_t = \ln(1 + R_t).$$

Since R_t follows a lognormal distribution then we can deduce that: $r_t \sim N(\mu; \sigma^2)$. Where μ it is the average and σ^2 is the variance for lognormal returns. The expected value of the composite return for a finite amplitude time interval $\mu - \frac{1}{2}\sigma^2$ is equal to the geometric mean of the realized returns rates in very small intervals of time. Prices P_t follow a stochastic process, and hence the differential equation of a stochastic process is:

$$dP_t = \mu \cdot P_t dt + \sigma \cdot P_t dz$$

$$\frac{dP_t}{P_t} = \mu \cdot dt + \sigma \cdot dz$$

$$\frac{\Delta P_t}{P_t} = \mu \cdot \Delta t + \sigma \cdot \varepsilon \sqrt{\Delta t}$$

Where z is a variable that follows a stochastic process such that: $\Delta z = \varepsilon \sqrt{\Delta t}$ and ε is a random variable with a distribution of N (0, 1). All this was detected by the Ito Lemma¹. $\mu \cdot \Delta t$ Gives the variable variation of the variable P_t in time, instead $\sigma \cdot \varepsilon \sqrt{\Delta t}$ it is a random element that characterizes the erratic movement of the variable, that is P_t , the variability P_t around the line. So we can write that returns can be calculated from the formula:

$$r = (\mu - \frac{1}{2}\sigma^2)(T - t) + \sigma \cdot \varepsilon \sqrt{T - t} . \text{ From the formula: } r_t = \ln(1 + R_t) \text{ finds that } = (1 + R_t) = e^{r_t} . \text{ We replace everything in the formula: } W_t = (1 + R_t)W_{t-1} \text{ and we will have:}$$

¹ Ito Lemma: Expressions referring to differential calculus, derivatives, integers studied for non-stochastic functions (no margin of error ...). There is also a calculation for stochastic processes and the derivatives are obtained thanks to the Ito lemma. In particular, if we know the stochastic process followed by x (dx = ...), the Ito lemma tells us what is the stochastic process followed by a certain function G (x, t), that is, allows us to derive the expression of dG = ...

$$VaR = \left[e^{(\mu - \frac{1}{2}\sigma^2)(T-t) + \sigma\varepsilon\sqrt{T-t}} \right] W_{t-1}$$

$$VaR = \left[1 - e^{(\mu - \frac{1}{2}\sigma^2)(T-t) + \sigma\alpha\sqrt{T-t}} \right] W_{t-1} \quad \text{With } \alpha \text{ the inverse of a normal } N(0, 1).$$

RESULTS

Introduction to the Dynamic Asset Allocation

Financial asset prices vary over time and the percentages of asset classes that compound a portfolio held by an investor can change over time. Let's assume we have a portfolio of three asset classes: short term, bonds and stock.

Asset Class	Short Term	Bond	Stock
Short term	40,0%	60,0%	0,0%
Conservative	25,0%	55,0%	20,0%
Moderate conservative	10,0%	40,0%	50,0%
Moderate aggressive	5,0%	30,0%	65,0%
Aggressive	5,0%	15,0%	80,0%

Table 1: Asset Class Profiles

The five profiles are classified according to the risk propensity. The following tables show the historical performance and historical volatility of the three asset classes that make up the portfolio. The Return and the volatility of the portfolio at time T = 0 are respectively 0.081 and 0.025279. This data is taken from Finantix datasets.

	Historical Asset Return	Historical Asset Volatility
Short Term	3.0%	1.0%
Bond	5.0%	10.0%
Stock	9.0%	20.0%

Table 2: Historical efficiency and asset volatility of asset classes

Suppose an investor wants to invest his money according to the own risk propensity, so that may have a certain amount of money at the end of a certain period. The initial investment is 1,100 euros, monthly savings of 200 euros, and the liquidity part the investor pays arrives at 40,000 euros. The goal that the investor wants to achieve, at the end of an investment period of 30 years, is 110,000 euros. The investor does not borrow any money during this period and expects to receive an amount of 10,000 euros after 7 years (receipt 1) and 1,000 euros in 4 years (receipt 2). The bold data is entered by the investor, so it can be modified. Other data cannot be changed. Investment Goal is calculated as the difference between Goal amounts And Borrowing).

Initial Investment	1100
Monthly Savings	200
Investment Goal	110000
Goal Amount	110000
Borrowing Amount	0
Investment Period	30
Return	0.081
Volatility	0.02527925

Table 3: Initial data

	Amount	Day	Month	Year
Liquidity	40000			
Receipt1	10000	11	2	2013
Receipt2	1000		6	2010
Receipt3	0		12	2020
Receipt4	0		7	2008

Table 4: Initial Data

Let's assume that the investor has a very high risk propensity. Therefore his profile is the "aggressive" one. Portfolio composition is: 5% short term, 15% bonds and the rest 80% in stock, as described in Table 1. During the investment period, the various asset classes will have a growth or a decrease, according to the financial market trend. Then, after the first year, the percentages of asset classes that make up the portfolio will change. At this point, the investor has two options:

- Do not do anything so as per Buy & Hold Strategy

- Redeem the asset class weights, as defined in the Constant Mix strategy

Choosing a strategy or another has implications on the value of the trend of the considered asset classes. At the end of the investment period, the investor is expected to receive a different amount of money according to the chosen strategy. If the asset class market has a well-defined upward trend, it is preferable Buy & Hold. At the end of the investment period, the percentages of assets will be changed and the investor risk profile will also change. In this way, you move from a risk profile with low performance and low risk to a profile that ensures a high return but at the same time also a greater risk. Otherwise, the market for these assets would have a downward trend in value, the return would be minimal, or worse there would be a loss. Constant Mix's strategy, on the other hand, logically acts counter-tendency. According to this strategy, the investor sells the assets of which the market has fallen, and buying those assets whose market has risen. This ensures the constant maintenance of asset percentages, thus also a constant risk profile. It often happens that the financial market is volatile and unpredictable, alternating years up and down. When the investor rebalances the asset weights might get better returns.

Buy & Hold Strategy

This section describes the Buy & Hold strategy more in details. The goal is to find the value of the portfolio held by the investor at the end of the investment period. The research is based on Excel tool. After the first year of investment, the portfolio is expected to have a higher value. The various activities that make up this portfolio do not grow to the same extent. Then, after the first year, portfolio compositions will change. To calculate the new percentages, it's used an application created with Excel. The initial value of the portfolio is € 43,500. The risk profile is aggressive. According to this profile, the investor invests 2,175€ in short term 6,525€ in bonds and 34,800€ in stock. To find out how much the portfolio of a single asset class is worth, we calculate how much are the monthly savings, initial investment and liquidity, and finally the part of the amount I expect to have in the future. The formula used is:

$$F = I \cdot e^{((\mu - \frac{\sigma^2}{2})T + A \cdot \sigma \cdot \sqrt{T})} + \sum_{i=0}^{(T)} S \cdot 12 \cdot e^{((\mu - \frac{\sigma^2}{2}) \cdot (i/12) + A \cdot \sigma \cdot \sqrt{(i/12)})} \pm \sum_{n=1}^4 FR \cdot e^{((\mu - \frac{\sigma^2}{2})(T-t) + A \cdot \sigma \cdot \sqrt{T-t})}$$

Where: I - initial investment plus the liquidity, S - Monthly savings, FR - are the future receipts, M - average portfolio yield, σ - volatility, A - the inverse of Normal (0.1), T - the investment period, t - future period, i-the months. The maximum number of cash inflows is n = 1,2,3,4. The above formula is calculated using Excel. The considered observed significance levels are 95% and 50%. As expected, at 50% the portfolio is worth than at 95%. In order to find the new

weights it is used the ratio between a portfolio of a single asset class and the sum of the three compiled portfolios of a single asset class for each year.

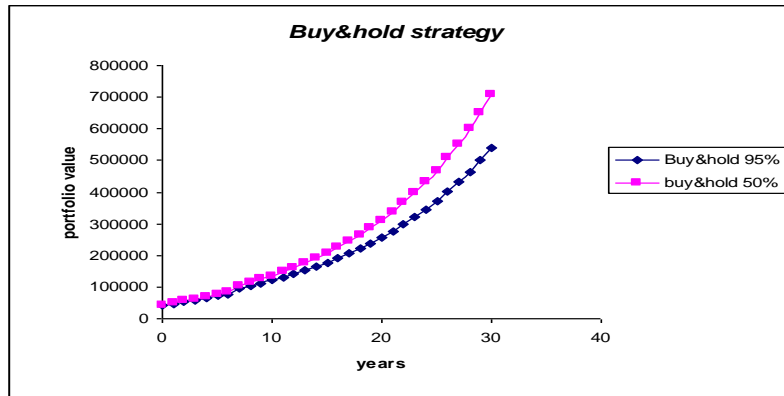
$$W_i = \frac{F_i}{\sum_{i=1}^3 F_i}$$

Observe that the weights at the end of the investment period have changed. The weight of short term bonds and bonds in the portfolio considered decreases during the investment period, instead the stock increases. At the beginning period the portfolio is composed of: 5% short term, 15% bonds and 80% stocks. Instead at the end of the investment period, the resulting weights will be 2.15% short-term, 4.95% bonds and 92.90% stocks at a 95% observed level of significance. Considering the level of observed significance 50%, the values are 1.52% short term, 6.13% bonds and 92.3 stocks. Short term bonds and bonds have declined, while stocks have risen. At this point you can find the average returns of the portfolio at the end of the first year, the second and so on for the whole period considered. The weights of the portfolio change every year so even the average yields change from year to year. The same is true for portfolio volatility. Both the average yield and the risk have both increased. Considerations are made on two levels of confidence because values of different levels of observed significance are used, 50% and 95%. At this point, the portfolio value of the three asset classes must be calculated taking into account that the percentages of the three assets change from year to year. As a result, the average yields and the average volatility also change as previously calculated. Excel applications are created to calculate the cash flow of all monthly savings, future receipts, which are the future revenue part that the investor expects to receive, initial investments and liquidity, which are respectively the part of the initial investment and liquidity that Make up the portfolio held by the investor. Calculations are made each year using the formula:

$$F = I \cdot e^{((\mu_i - \frac{\sigma_i^2}{2}) \cdot T + A \cdot \sigma_i \cdot \sqrt{T})} + \sum_{i=0}^{(T)} S \cdot 12 \cdot e^{((\mu_i - \frac{\sigma_i^2}{2}) \cdot (i/12) + A \cdot \sigma_i \cdot \sqrt{(i/12)})} \pm \sum_{n=1}^4 FR \cdot e^{((\mu_i - \frac{\sigma_i^2}{2}) \cdot (T-i) + A \cdot \sigma_i \cdot \sqrt{T-i})}$$

Note that average returns and average volatility change from year to year, according to the year in consideration. Calculations are made for two observed levels of significance, at 50% and 95%, the last a more prudent measure. So, considering the Buy&Hold strategy that does not modify the percentages of asset classes per year, there is a portfolio of €43,500, which at the end of the investment period will be €70,803.6136. Otherwise using a more prudent measure (95%) €540,597.9952. The chart shows the comparison between the two levels of significance

observed, for the Buy&Hold strategy. In abscissas there is the investment period, and the ordinate has the value of the portfolio. Charts follow an exponential trend. Portfolio value grew more in the last decade.



Graph 1: Portfolio according to the BUY & HOLD strategy

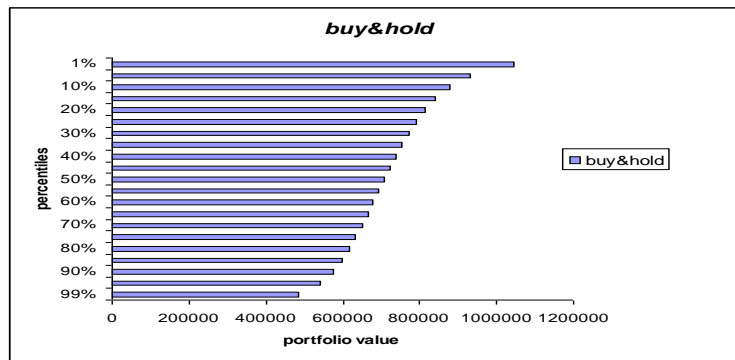
The following chart shows the value of the portfolio being considered for each probability level, from 1% to 99%.

Percentiles	Portfolio Value
99%	485,456
90%	574,071
80%	616,631
70%	649,471
60%	679,051
50%	708,032
40%	738,362
30%	772,380
20%	814,358
10%	876,691
1%	1,046,152

Table 5: Probability to Reach the Goal

So considering the Goal Amount set as the investor's goal, one can find the likelihood to achieve the goal. For example, with a probability of 30%, you may be able to have an amount of

€772,379.7 euros at the end of the investment period and a 99% probability of having an amount of €485,455.7. It is noted that the investment goal set as a target is 110,000€. It is thus clear from the graph that the investor can safely reach the target at a probability of 99%.



Graph 2: Probability to reach the goal amount

Constant Mix Strategy

Let's consider again the investor's portfolio, with the same data as in the Buy& Hold strategy. The portfolio consists of three asset classes as previously, and even the risk profile is always the same. This strategy, as mentioned before, shows the percentages at the starting level, so it rebalances every year the weights of asset class of the portfolio into account. To rebalancing the weights, the investor sells those assets which the market has gone up and buys those assets which market has dropped. At this point we calculate the monthly savings that the investor pays each month. At the actualized sum of these revenues, we add the initial investment and liquidity, actualized. The formula used to make these calculations is:

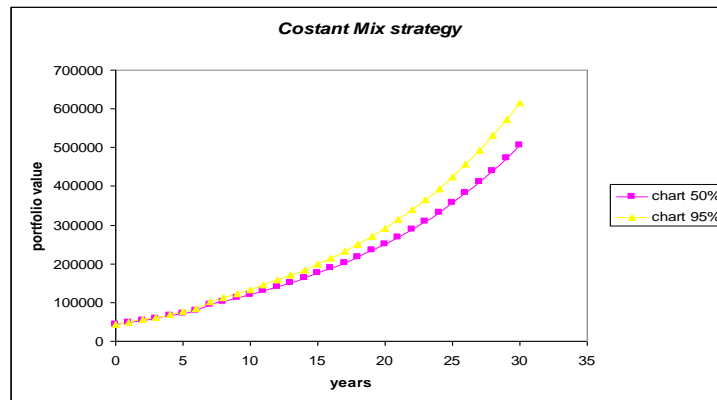
$$F = I \cdot e^{((\mu - \frac{\sigma^2}{2}) \cdot T + A \cdot \sigma \cdot \sqrt{T})} + \sum_{i=0}^{(T)} S \cdot 12 \cdot e^{((\mu - \frac{\sigma^2}{2}) \cdot (i/12) + A \cdot \sigma \cdot \sqrt{(i/12)})} \pm \sum_{n=1}^4 FR \cdot e^{((\mu - \frac{\sigma^2}{2}) \cdot (T-t) + A \cdot \sigma \cdot \sqrt{T-t})}$$

The formula shows us that returns and volatility do not depend on time, unlike the Buy & Hold strategy. Calculations are made for 50% and 95% significance levels, same as in the previous strategy. At the end of the investment period, the portfolio will have a value of 616945.2 euro at a level of 50% and a value of 506,081.262 euro, taking into account the level of significance of 95% .The last one is a more prudent measure.

years	95%	50%
0	43,500	43,500
5	71,501	77,252
10	120,709	133,764
15	175,606	199,760
20	251,049	292,488
25	356,513	425,028
30	506,081	616,946

Table 6: CONSTANT MIX Portfolio

The chart gives me the differences in the portfolio in consideration of the two levels of significance observed. As mentioned the two charts have an exponential trend. The value of the portfolio at the 95% observed significance level has lower values comparing to the line with a significance level of 50%. That is true for each year within the investment period.



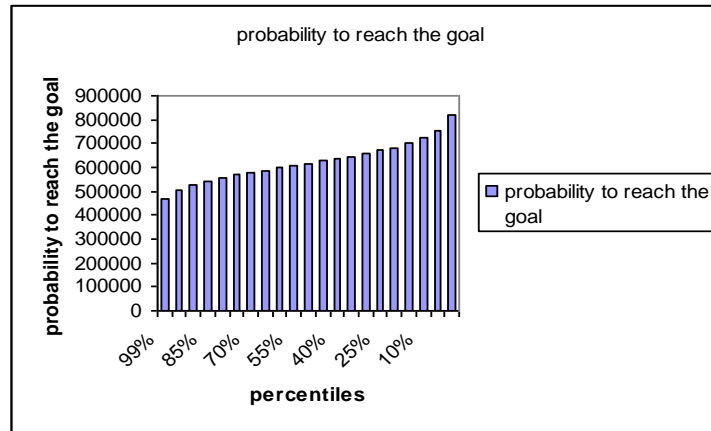
Graph 3: Portfolio according to Constant Mix strategy

The below table below shows the data that the portfolio has for each probability level, so from 1% to 99%.

Probability to reach the goal	
Percentiles	Portfolio value
99%	466,790
90%	528,525
80%	557,194
70%	578,933
60%	598,249
50%	616,946
40%	636,285
30%	657,712
20%	683,794
10%	721,846
1%	821,722

Table 7: Probability to reach the goal

The following chart shows the value of the portfolio considering every probability level, from 1% to 99%. So considering the Goal Amount set by the investor, one can find the likelihood to achieve this goal. For example, with a probability of 30%, it may be able to have at the end of the investment period an amount of €657,712.3 Also, with a probability of 99% to have an amount of €466789.9. If the investor wants to get an amount of about €578,930 then the probability that he achieves this goal is 70%. Notice that the investment goal is set at €110,000. It can easily be verified from the graph that the investor can safely reach the target at a probability of 99%.



Graph 4: Probability to reach the goal amount

DISCUSSION

The difference between the two strategies consists in the weights that compound a portfolio. The percentages of asset classes that make up a portfolio can change from year to year in the case of Buy & Hold, or remain predefined as per investor's decision, Constant Mix's strategy. So, in order to find the differences, we start with the comparison between the weights of the portfolio of the two strategies. Table 1 shows that portfolio composition: 5 % short term bonds of 15% and 80% stocks. In Constant Mix strategy these percentages remain the same for the entire duration of the investment period. Instead, in Buy & Hold strategy, the weights change from year to year. In time the investor will have a lower weight for short term bonds and a greater weight for stocks. Table 8 shows the performance and volatility values for the Buy & Hold strategy. The values change from year to year. It starts from an average return on the portfolio of 0.081, and then has a higher average return of 0.086. The same is true for risk as well; it starts with a risk of 0.025279 to reach 0.033888 at the end of the investment period. Instead, for the Constant Mix strategy, performance and volatility remain the same throughout the investment period. By comparing the performance and the average risk of the two strategies, the Buy & Hold strategy will have a higher yield and risk than the Constant Mix strategy.

Years	portfolio return		portfolio risk	
	95%	50%	95%	50%
0	0,081	0,081	0,025279	0,025279
5	0,082582	0,082318	0,027781	0,027133
10	0,083463	0,083223	0,029124	0,028462
15	0,084406	0,084198	0,030588	0,029946
20	0,085253	0,085091	0,031928	0,031352
25	0,08603	0,085906	0,033181	0,032677
30	0,086734	0,086636	0,034338	0,033896

Table 8: Historical return and volatility

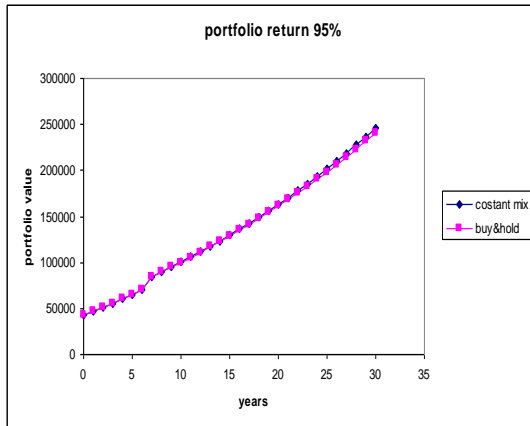
At this point, after comparing the weights, returns and risks we can draw conclusions about the value of the portfolio. A high return value implies a high value of the portfolio in consideration. Consequently, for the Buy&Hold strategy we will have a portfolio amount higher than that of the other strategy. In the following table, note that the value of the portfolio for each strategy considered is equal to €43500. From year to year the value of the portfolio increases and we see that the value of the portfolio at a level of significance observed is 50% greater in the case of the Buy & Hold strategy. Same thing also happens for a more cautious 95% significance level.

Years	BUY&HOLD		COSTANT MIX	
	95%	50%	95%	50%
0	43,500	43,500	43,500	43,500
5	71,393	77,648	71,501	77,252
10	120,900	135,875	120,709	133,764
15	177,309	206,869	175,606	199,760
20	256,880	311,172	251,049	292,488
25	371,714	468,050	356,513	425,028
30	540,598	708,032	506,081	616,946

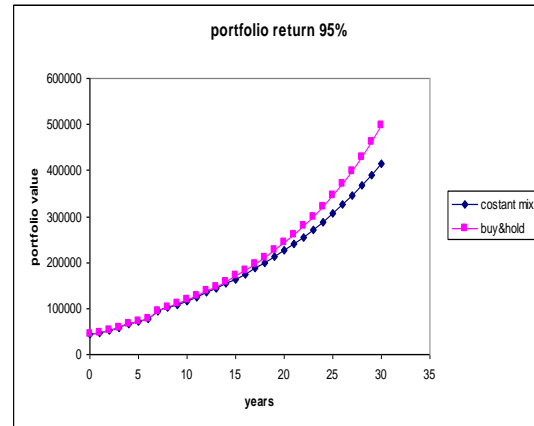
Table 9: Portfolio value, BUY & HOLD and COSTANT MIX strategies

CONCLUSIONS

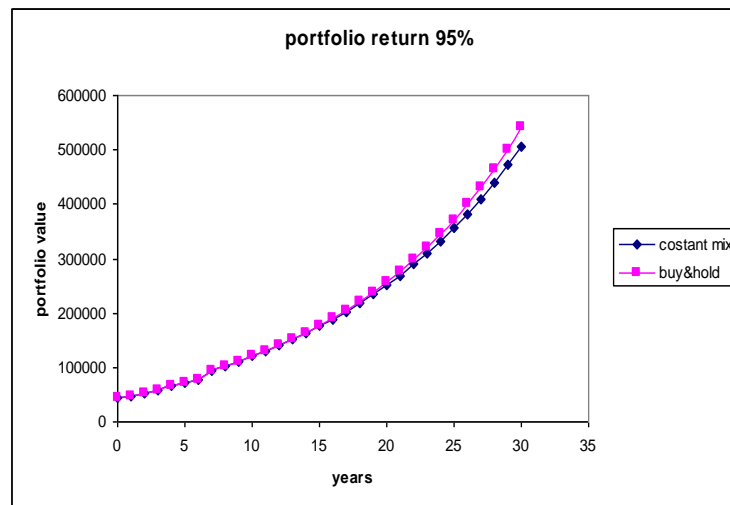
This section describes how the value of the portfolio changes in the two strategies according to the investor risk propensity. If the investor has a very low risk aversion, so he does not want to risk (low volatility, low return) the risk profile he will choose will be a "short term profile" with a 40% short-term percentage composition 60% bonds. The portfolio at the end of the investment period will be very low compared to what is in the case of an "aggressive" profile. On the other hand, an investor with a moderate conservative profile will have a portfolio compound by 10% short-term, 40% bond and 50% equity stock. The portfolio will have an intermediate value between the portfolios that are obtained in the case of "short term" and "aggressive" profiles. By comparing the two strategies in the case of "Short Term Profile", we see that the differences are just remarkable. Over the last decade the values are bigger for the Constant Mix strategy. In this case, you should use the Constant mix strategy. For the other two profiles you see that the highest value is in the case of the Buy & Hold strategy. The differences between the two strategies are more noticeable in the case of a "moderate conservative profile"



Graph 5: Short Term Profile



Graph 6: Moderate Conservative Profile



Graph 7: Aggressive Profile

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