
FORMULATION OF PROBABILISTIC DIFFERENTIAL EQUATIONS USING IMAGE TECHNOLOGY IN NO DATA PROBLEM

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ABSTRACT

In Fuzzy-Bayes decision making rule, we consider two distributions of subjectivity distribution which expresses human subjectivity and membership function which converts and maps state of nature into fuzzy event by membership function. The membership function is a kind of filter function that is set by each decision maker and transforms and maps the natural state into fuzzy events with membership functions. As this filter function is set by the subjectivity of each decision maker, consequently giving a certain degree of freedom to fuzzy events. With Zadeh, a fuzzy set was proposed, and Tanaka et al extended it to a decision-making problem. This extension is called fuzzy Bayes decision making rule, but it was shown by Hori that it is included in the subjective modification of the Wald function. This indicates that fuzzy · Bayes decision making rule is included in subjective Bayes theory. However, Hori has mentioned that Type 2 fuzzy is unique in fuzzy-Bayes decision making law. In addition, Markov decision process in the fuzzy event after the mapped and transformed the state of nature by the decision-making membership function, subjectivity after mapping the state of nature by subjective distribution and utility function Markov decision process in utility was derived. In this paper, the subjective view is a subjective distribution of the state of nature, after conversion and mapping, and utility regards the state of nature as a utility function after conversion and mapping, and map technique in the expansion principle of The mapping We formulates the stochastic differential equations based on. Furthermore, as a mapping of the mapping, fuzzy theory formalizes the stochastic differential equations based on the subjective distribution and the utility function in the fuzzy event as type 3 fuzzy. Here, it is considered that solutions of these stochastic differential equations follow Ito's integral. As a future subject, I would like to study the effectiveness and validity of fuzzy theory by actually guiding the difference in optimal behavior when not introducing fuzzy logic theory from Ito integral and Max product law. In particular, by this formulation, when the membership function is an identity function, the probability differential equations are equal when introducing fuzzy logic and not introducing it. This proves that fuzzy

logic is one type of subjective Bayes theory when the decision maker is a dangerous neutral person

Keywords: Mapping technique, stochastic differential equations, Ito integral, type 2 fuzzy, type 3 fuzzy

1. INTRODUCTION

Ito [1, 2] shows the solution of probability differential equation (1). This is what is called so-called Ito integration. Here, the first term of the expression (1) represents the transition of the average, and the second term represents the transition of the standard deviation and the noise term.

$$\frac{dS}{dt} = b(t, St) + \sigma(t, St) \cdot Wt \quad (1)$$

With Zadeh, a fuzzy set was proposed, and Tanaka et al extended it to a decision-making problem [3]. This extension is called fuzzy Bayes decision making rule, but it was shown by Hori et al [4, 5] that it is included in the subjective modification of the Wald function [6]. In addition, Hori et al [7, 8] mapped the state of nature with the decision distribution process and the utility function of Markov decision process and fuzzy event in the fuzzy event after mapping / transformation of the state of nature by the membership function of the decision maker Markov decision process on later subjectivity and utility was derived. In this paper, the subjective view is a subjective distribution of the state of nature, after conversion and mapping, and utility regards the state of nature as a utility function after conversion and mapping, and as a mapping technique in Zadeh's extension principle we formulate probabilistic differential equation based on. Furthermore, as a mapping of maps, we formulate probabilistic differential equations based on subjective distribution and utility function in fuzzy events as fuzzy theory type 3 fuzzy.

2. Flow of fuzzy phenomenon research until now

Tanaka et al formulated the fuzzy-Bayes decision making rule by integral transformation based on the expected utility maximization theory as an extension to Wald's subjective modification fuzzy event. Hori et al formulated the fuzzy · Bayes decision making rule which extended Wald's decision function to fuzzy OR combination and fuzzy AND combination with many subjective distribution. This decision-making law is based on the state of nature

Is a decision-making rule after mapping and conversion to fuzzy events, and it is an OR type 2 fuzzy by mapping fuzzy functions such as subjective distribution and utility functions to fuzzy events. Furthermore, Hori introduced the Markovian time concept to the state of nature, and derived the Markov process and Markov decision process in fuzzy events. This is a natural

extension to the stochastic process theory of Wald's decision function, and the fuzzy event of appearance of the natural state becomes a Markov process having a fuzzy transition matrix, and as a result of the Monte Carlo simulation, the annihilation, reversal, resurrection Repeat the cycle. Finally, Hori et al proposed an illusion state identification method as an example of adaptation of these fuzzy / Bayes decision making rules. In addition, Hori et al. Firstly used the max product method by mapping / transformation of membership functions of fuzzy events in a fuzzy event in which the subjective distribution and utility function in the no data problem transit like ergodic Markov We formulated these Markov decision processes. Note that this series of flows is a natural extension to the stochastic process of Wald's decision function. Next, we consider subjective distribution and utility function as fuzzy functions, subjectivity maps / converts natural state by subjective distribution, utility assumes that natural state is mapped / converted by utility function, The subjectivity and the utility also showed that it follows Markov process. Finally, the subjectivity and utility in fuzzy events were propagated by Markov processes in which each element of the transition matrix follows the Markov process, and proposed the Markov decision process by the Max product method.

3. Formulation of stochastic differential equations by mapping technique in no data problem

In equation (1), the subjective S_t is considered to be a map of the natural state S_t mapped and transformed by the subjective distribution $\Pi_t(S_t)$. In addition, the subjective distribution is identified piecewise linearly by lottery, and it is a fuzzy OR bond. Therefore, the extension principle of Zadeh can be applied, and the subjectivity can be formulated by stochastic differential equations as follows.

$$\begin{aligned} \frac{d\Pi}{dt} &= \sup_{\{Z_t = \Pi_t(S_t)\}} b(t, S_t) + \sigma(t, S_t) \cdot W_t \\ &= b(t, \Pi_t^l(Z_t)) + \sigma(t, \Pi_t^l(Z_t)) \cdot W_t \end{aligned} \tag{2}$$

Likewise, since the lower utility function $U_t(S_t|D)$ given the decision D is also identified by lottery, it can be regarded as a fuzzy OR combination, fuzzy mathematics is applied and the utility U_t is It can be formulated by stochastic differential equations.

$$\begin{aligned} \frac{dU}{dt} &= \sup_{\{Z_t=U_t(S_t|D)\}} b(t, S_t) + \sigma(t, S_t) \cdot W_t \\ &= b(t, U_t^{-1}(Z_t|D)) + \sigma(t, U_t^{-1}(Z_t|D)) \cdot W_t \end{aligned} \tag{3}$$

In the no data problem, both subjectivity and utility are fuzzy OR combinations, so the decision making rule becomes a high risk high return problem of subjectivity and utility, and it is defined by the Max product method [9] by the following equation. Here, it is assumed that equations (2) and (3) are solved by Ito integral.

$$\max_D \max_{S_t} \Pi_t(S_t) \cdot U_t(S_t|D) \tag{4}$$

Equations (2) and (3) above can be considered Type 2 fuzzy. Furthermore, we introduce the concept of fuzzy events to this decision-making problem. In this case, taking subjectivity and utility in the fuzzy event after mapping and converting the natural state by the decision-making membership function $\mu_t(S_t)$, the subjectivity in the fuzzy event can be expressed as fuzzy The utility in the event is given by equation (6)

It is formulated as stochastic differential equations. Here we note that these are type 3 fuzzy.

$$\begin{aligned} F\Pi_t(Z_t) &= \sup_{\{Z_t=F\Pi_t(S_t)\}} b(t, S_t) + \sigma(t, S_t) \cdot W_t \\ &= b(t, \mu_t(\Pi_t^{-1}(Z_t))) + \sigma(t, \mu_t(\Pi_t^{-1}(Z_t))) \cdot W_t \end{aligned} \tag{5}$$

$$\begin{aligned} FU_t(Z_t|D) &= \sup_{\{Z_t=FU_t(S_t)\}} b(t, S_t) + \sigma(t, S_t) \cdot W_t \\ &= b(t, \mu_t(U_t^{-1}(Z_t|D))) + \sigma(t, \mu_t(U_t^{-1}(Z_t|D))) \cdot W_t \end{aligned} \tag{6}$$

Next, solving equations (5) and (6) with Ito integral, it is a high risk high return problem, so according to the Max product method, these decision rules are defined by the following equations.

$$\max_D \max_{S_t} F\Pi_t(S_t) \cdot FU_t(S_t|D) \tag{7}$$

Here, when the membership function is an identity function, equations (2), (5), (3) and (6) have the same value,

The optimal behavior obtained by Eq. (7) is the same. This proves that fuzzy theory is included in the subjective Bayesian theory regardless of arbitrary operation when the decision maker is a dangerous neutral person. In the future, in the case where the membership function is convex upward (the decision maker is a dangerous person) and the case where the membership function is convex downward (decision maker is a risk avoider), actually two different probability derivatives We are going to solve the equation and consider each optimal behavior.

4. Conclusions

In this paper, the conventional stochastic differential equations are considered fuzzy by introducing the state of nature, the probability differential equation using the mapping technique is Type 2 fuzzy, and furthermore, the stochastic differential based on the fuzzy event using the mapping technique We considered that the equation is type 3 fuzzy. As a future task, we aim to build a Markov decision process in fuzzy events in the no data problem by actually solving equations (2), (3), (5) and (6) with Ito . Finally, we hope that our efforts can contribute to future artificial intelligence research by clarifying the relationship between type 2 fuzzy and type 3 fuzzy and stochastic differential equations.

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