THE MODERN THEORY OF ECONOMIC DEVELOPMENT

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ABSTRACT

This article looks for an efficient fiscal policy leading to the optimal growth path in developing countries where human capital accumulation is an engine of economic growth. We find that there exist a threshold where fiscal policy becomes detrimental for economic development and as long as it is not attained, fiscal policy is powerful to enhance economic growth otherwise, knowledge increase policy through optimal fiscal policy blocks the development prospects. Indeed, the study’s results join those found in the literature of the relationship between growth and fiscal policy where they move in the opposite direction after the threshold crossed. Consequently, the study highlights fiscal policy limitations in development economics target.

Keywords: optimal fiscal policy, optimal growth path, fiscal policy threshold, economic development

JEL Classification: E13, E62, F62, F63

1. Introduction

Endogenous growth theory emergence came from the difficulties both to prove growth sources and increasing returns introduction in the competitive dynamic optimization model. Therefore, growth theory experienced a boom in the middle of the years 1980s since it was found that, knowledge and human capital are the main growth sources in a given country and increasing returns can hold in a neoclassical competitive growth model (Romer, 1986; Lucas, 1988; Azariadis-Drazen, 1990) and yields to a great body of literature on endogenous growth which covers almost all the fields in economics. More precisely, Robert Lucas Jr, 1988 article aim was to establish human capital as the main source of comparative economic development on the basis of the Solow (1956) neoclassical growth model in which he introduced human capital component initiated by Schultz (1960) and Becker (1964) in order to render growth endogenous through human capital which turn out to be a mechanics of economic development. But dealing with education as a mechanics of economic development, yields finally to notice that, there are inequalities in education system since its access is limited in the market because of parental social status differentials in the society since human capital accumulation is costly and deserve parental altruism for quality to be preferred to quantity in children procreation process. Therefore, since developing countries economic retard is partly explained by insufficient
investment in human capital and not really in R&D yet, because those countries still far from their development frontier for it to be profitable (Acemoglu et al, 2005), unfortunately both development economists pioneers and endogenous growth theorists forgot to propose fiscal policy tools as the way to get developed since it may support incentives to invest in human capital accumulation. Quantity choice preferences because of cultural believes such that children are future resting income providers for example, raises questions on the way quality may supplement quantity for development to take-off. Therefore, how to make skills increase in that context?

In order to provide an answer to that question, this article is an essay where fiscal policy is chosen to experiment human capital financial support, thus its increases for development emergence in least developed economies to be effective. Indeed, the government intervention to supplement the market inefficiency is welcome and necessary for that goal to be achieved. According to Romer (1990) the growth rate increase is a positive function of human capital but not of population size. Consequently, incentives to invest in human capital accumulation are crucial for development target. Therefore, waiting for financial aids from high income countries and/or the world organizations programs is limited, this article proposes to see in how far fiscal policy can yields human capital increase in developing countries in order for those countries to get developed faster since funds for education can be obtained from home agents income. After the crisis of development economists pioneers analysis held between the years 1970s and the 1990s, economists looked again at those of ideas with fresher eyes and recognize them to have finally a sense after all (Krugman, 1994), thus makes the theory regained interest in the beginning of the years 1990s (Murphy-Shleifer-Vishny, 1989) almost after the debt crisis in development economies which led those countries allow IMF and the World Bank to conduct adjustment structural policies for macroeconomic stability, then empirical studies of the government action in developing world emerge since the years 1990s. Therefore, whereas, in the concern of growth theorists, fiscal policy relationship to growth began in the years 1980s with the works of Eaton (1981), Barro (1990), Jones and Manuelli (1990, 1992), King and Chamley (1981), Judd (1987, 1990), King-Rebelo (1990), Lucas (1990) and Yuen (1990) and mostly conclude to a negative relationship between growth and fiscal policy. In developing studies, three approaches can be distinguished in the relation between fiscal policy and economic development. The first approach highlights a sharp rise in social spending that occurred in

1 Hirschman, 1958; Leibenstein, 1957; Lewis, 1954; Myrdal, 1957; Nelson, 1956; Rosenstein Rodan, 1943) remains widely open where we can find the following notions: the Big-Push (Rosenstein Rodan, 1943), the Economic Dualism (Lewis, 1954), the Stages of Economic Development (Rostow, 1960) and The Strategy of Economic Development (Hirschman, 1958)
developing economies, the second approach studies taxation effect on growth and the third approach is focused on traditional Sector such as land and inequality along the development process. According to the first approach, Lindert (2004) discusses two things, the first is why the growth of government does not slow economic growth, as the government becomes larger, policymakers take more care to tax in ways that minimize distortions. The second thing is that, some policies have reduced employment (welfare, unemployment compensation, and public pensions) and in doing so have removed workers with below average productivity from the workforce, resulting in higher labor productivity. For the second approach, several studies have found a strong negative correlation between the relative size of the agricultural sector and the relative size of government, other things constant (Burgess and Stern (1993), Peltzman (1980), Stotsky and WoldeMarian (1997), and Tanzi (1991)). In fact, the studies find that the relative size of the agricultural sector is more closely correlated with the relative size of government than are other indicators of development, such as income per capita. There is a related literature where the concern is that taxation and other government policies push economic activity underground (Schneider and Enste (2000, 2002); Schneider (2007)). In this literature, the paper is Loayza (1996) which looks at how government policies contribute to the informal sector and how the informal sector affects long-run economic growth in an endogenous growth model. His theoretical and econometric analysis indicates that the informal sector reduces both public infrastructure investment and economic growth. However, while Loayza focuses on the urban informal sector and long-run balanced growth, Mourmouras and Rangazas (2008) focus on the rural informal, or traditional, sector and the structural transformation in a framework that allows for transitional growth, where the size of the traditional sector, economic growth rates, and tax rates all change over time. However, Mourmouras and Rangazas (2008) is interested in the interaction between transitional growth and the setting of tax rates when the traditional sector is difficult for the government to tax. They find that, the reduction in the tax base, when tax rates are raised, is directly proportional to the relative size of the traditional sector. In this sense, tax evasion is more of a problem for developing economies—even on the margin. When the traditional sector is relatively attractive in general, it gives households a “legal” way to avoid taxes. When the traditional sector is not generally attractive, households and firms will remain in the modern sector and must illegally avoid taxes or create more complicated legal ways of avoiding taxes. The third approach is the one which deals with land inequality and development, a growing literature that suggests that land inequality may hamper growth and reduce growth through several mechanisms, including its effect on economic and political institutions, influence over agricultural policy, credit market development, and support for public schooling (Erickson and Vollrath (2004)). A common feature of these mechanisms is the attempt by politically powerful interests (such as landowners) to retain a low-cost work force in agriculture by limiting the options of workers outside of agriculture. Hayashi and. Prescott (2006) review patterns of
Japanese economic development and argue that the Japanese miracle did not take place until after World War II because of barriers that kept agricultural employment constant throughout the prewar period. They develop a two-sector neoclassical growth model in which the resulting sectoral misallocation of labor creates disincentives for capital accumulation that accounts well for the depressed output level in interwar Japan. They also explain the existence of the labor barrier by appealing to the prewar Japanese tradition of patriarchy that forced the son in each family who was designated as her to stay in agriculture. In Mourmouras and Rangazas (2008), landowners may be able to maintain a low-cost work force by supporting high tax rates on modern production sectors where incomes are easier to identify and tax. High taxes levied on modern production techniques act as a barrier, favoring traditional agriculture, especially when production and payment methods there are informal. Workers avoid high tax rates by staying in the traditional sector, driving down wage rates there to the benefit of landowners.

This paper looks for optimal fiscal policy for long run growth establishment which ensure development take-off since human capital funds can be obtained through fiscal policy. Two steps are used, the first step doesn’t consider physical capital influence on the long run growth yet in constrast to the second approach where it is considered. Unfortunately, the results found join the endogenous growth studies findings where fiscal policy is detrimental for growth, thus for development in the case of this study. Indeed, the afford made in this paper in order to highlight funds provision for incentives to accumulate human capital revealed finally, that this is not a suitable economic policy for that purpose. Indeed, inequalities found in the education sector access, can only be alleviated not really eliminated because as taxes rise, optimal growth path begin to be far from its long run frontier of economic development.

The article looks for optimal fiscal policy leading to an optimal development path in two cases where the first is achieved when physical capital is not yet included in the human capital accumulation policy purpose, thus show-off an optimal path where the economic growth rate remains not links to fiscal in the both cases and in opposite, time spent in education sector turns out to depend to fiscal policy after the cross of the threshold level of taxation, then fiscal policy became detrimental for economic development. Between intensive policy and non intensive taxation policy, a threshold exists through which the expected results became negative for the purpose of achieving development through human capital increase. The optimal growth path is a four variable vector composed of the economic growth rate, $g$; time spent in education sector, $u$; the ratio of physical to human capital, $k$ and the ratio of consumption and physical capital, $w$; find that, the taxation policy before the reach of the threshold increases $u$ which doesn’t directly depends to it, increases also $k$ and $z$ which means that, its effect is positive for long run economic
development. In contrast, after the threshold crossed, fiscal policy links with education which was not the case before and makes fiscal policy decreases both $k=K/h$ and $z=c/K$ which means that, eviction is created, the government policy is render population poor and enable to invest in something in general. Therefore fiscal policy power remains as long as a given threshold is not crossed. Thus highlights the difficulty for public education alone to found development which must be added to private education and look for the way to decrease inequality in education access only.

The scientific contribution of the analysis: holds on several aspects which are, the introduction of fiscal policy in endogenous growth models in development study. It enlarges the discussion on development though human capital increase possibility in a the aspect of funds provision. Thus claims quantity decrease in the choice of children for quality to increase through altruism in human capital support process.

The article is exposed as follows: section2 presents the basic model, section3 studies the equilibrium path in non intensive case, section4 re-expose the problem in intensive physical capital case and finally section5 concludes on the analysis.

2. The economic growth model with endogenous fiscality

2.1 The consumer’s behavior

The intertemporal utility function of the representative agent living indefinite time is given by equation (1) such that:

$$
\int_{t=0}^{\infty} e^{-\rho t} \frac{c(t)^{1-\sigma}}{1-\sigma} dt 
$$

(1)

Where $c(t)$ is consumption, $\rho > 0$ is the discount rate et $\sigma > 0$ is the inverse of the elasticity of intertemporal substitution.

For simplicity, we assume that the economy doesn’t issue debt, therefore according to that hypothesis, the consumer’s budget constraint can be written such that equation (2) i.e

$$
c(t) + s(t) = (1 - \tau_k)r(t)K(t) + (1 - \tau_h)w(t)[u(t)h(t)] + T(t)
$$

(2)

The consumer pays a tax on physical capital hold, $\tau_k$ as well as on human capital, $\tau_h$ where $r(t)$ is the return rate on physical capital, $w(t)$ is the return rate on human capital, $T(t)$ are public transferts et $u(t)$ is time spent to work duty.
2.2 The human capital evolution law of motion

The representative agent can increase its human capital stock, $h(t)$ through time spent variable increase, $1-u$ in the education sector. We assume human capital evolution law to be written such that equation (3) i.e

$$h(t) = A(t)[1-u(t)] - \delta_h h(t)$$

(3)

Where $A>0$ is the marginal productivity of the education sector and $1-u(t)$ is time spent in the education sector where human capital depreciate at a positive rate, $\delta_h >0$

2.3 The behavior of the firm

The firm produces goods using a constant return technology expressed by equation (4) such that

$$F(K(t), h(t)) = BK(t)^\mu [u(t)h(t)]^{-\mu}$$

(4)

Where $B>0$ is the marginal productivity of good and services sector, $0 < \mu < 1$ is the elasticy of production according to physical capital.

Profit maximization implies that, factors are remunerated at their margin productivity expressed by equations (5) and (6) such that :

$$r(t) = \mu BK(t)^{\mu-1} [u(t)h(t)]^{-\mu}$$

(5)

$$w(t) = (1-\mu)BK(t)^\mu [u(t)h(t)]^{-\mu}$$

(6)

2.4 The physical capital law of motion

The consumer spends his income in consumption goods, $c(t)$ and in physical goods, $s(t)$ therefore, physical capital law of motion can be written such that equation (7) i.e

$$\dot{K}(t) = s(t) - \delta_k K(t)$$

(7)
Where $\delta_k > 0$ is physical capital depreciation rate

### 2.5 The good production market equilibrium

The good market equilibrium satisfies equation (8) i.e

$$c(t) + s(t) = BK(t)^\mu [u(t)h(t)]^{1-\mu}$$

### 2.6 The government budget constraint

At each period, the government budget constraint verifies the following equation

$$\tau_k r(t)K(t) + \tau_h w(t)[u(t)h(t)] = T(t)$$

Where $T$ are transfers the government gives to the representative agent

### 3. The competitive equilibrium

The government policy consists on the determination of optimal fiscal policy leading to the optimal growth path where two measures called taxes on physical capital, $\{(1 - \tau_k) r(t) - \delta_k\}$ as well as on human capital, $\{(1 - \tau_h) w(t) - \delta_h\}$ are used to achieve the equilibrium. Those measures are respectively « the return rate of physical capital » and « the return rate of human capital accumulation » and making them move as we’ll show it yields to optimal fiscal policy first and the corresponding optimal growth path after. Through which we examine the powerful of the optimal policy on development.

#### 3.1 Caracterization of the competitive equilibrium

The dynamic programming principle is based on the Hamiltonian resolution such that :

$$H(K, h, c, u, \theta_1, \theta_2) = \frac{c^{1-\sigma}}{1-\sigma} + \theta_1 [BK^\mu (uh)^{1-\mu} - c - \delta_h h] + \theta_2 [A((1-u)h) - \delta_h h]$$
The first order conditions of the dynamic programming are given by equations (10) and (11) such that:

$$\frac{\partial H}{\partial c} = 0 \quad \Rightarrow \quad c(t)^{-\sigma} = \theta_1$$

(10) means that total available income must be allocated between consumption goods and physical goods. Meaning that, the marginal consumption price must equalize physical capital price, $\theta_1$

$$\frac{\partial H}{\partial u} = 0 \quad \Rightarrow \quad (1 - \mu)BK^{\mu}(uh)^{-\mu} \theta_1 = A\theta_2$$

(11) means that, total available time must be allocated between the education sector and the production sector. Meaning that, at the margin, the workforce income must be equal to human capital accumulation cost, $\theta_2$

Physical capital price moves according to equation (12) i.e:

$$\dot{\theta}_1 = \rho \theta_1 - \frac{\partial H}{\partial K}$$

(12)

Human capital price moves according to equation (13) i.e:

$$\dot{\theta}_2 = \rho \theta_2 - \frac{\partial H}{\partial h}$$

(13)

The transversality conditions respectively for physical capital and for human capital are given by equations (14) and (15) i.e

$$\lim_{t \to \infty} \left[ e^{-\rho t} K(t) \right] = 0$$

(14)

$$\lim_{t \to \infty} \left[ e^{-\rho t} h(t) \right] = 0$$

(15)

3.2 The stationary growth path
Definition 1: the stationary growth path is the locus on the space where consumption and capital variables move at the same growth rate whereas time spent in the education sector remains constant. Indeed, according to definition 1, along the stationary path, we have:

\[
\frac{c(t)}{c(t)} = \frac{K(t)}{K(t)} = \frac{h(t)}{h(t)} = g \quad \text{et} \quad \frac{u(t)}{u(t)} = 0 \quad (16)
\]

Definition 2: the stationary equilibrium is the locus on the space where \( k = K/h \) equals \( z = c/K \)

Proposition 1: according to the model and definitions 1 and 2, there exist a unique fiscal policy couple defined such that equations (17) and (18) i.e

\[
1 - \tau_k = \left( \frac{B}{\phi_{kk}} \right)^{1-\mu} \quad (17)
\]

\[
1 - \tau_h = \left[ \frac{\phi_{kh}}{\phi_{kh}} \right]^{\mu(1-\mu)/1-\mu+\mu^2} \quad (18)
\]

(See the appendix for proof)

Proposition 2: according to proposition 1 and definition 2, the economic system admits a unique optimal growth path defined by a vector of 4 variables such that equations (19)-(22) i.e

\[
g = \frac{A - (\rho + \delta_h)}{\sigma} \quad (19)
\]

\[
u = \frac{\rho + (1-\sigma)(\delta_h - A)}{\sigma A} \quad (20)
\]

\[k = (1 - \tau_k)^{1-\mu} \phi_k \quad (21)\]
\[ z = B(1 - \tau_k)\phi_z^{-\frac{1}{2}} - \phi_z^{-\frac{1}{2}} \quad (22) \]

(See the appendix for proof)

We can see that, \( u \) and \( g \) depend on the marginal productivity of the education sector, \( A \) and the inverse of the elasticity of substitution, \( \sigma \) such that, they increase, 1-\( u \) as well as \( g \). In contrast, \( k \) and \( z \) are linked to fiscal policy such that it increases \( h \) and \( c \) but decrease physical capital prospects in the first step. Because the increase of the wage rate income yields consumption demand increase as well as the demand of education in response to the supply of education by the social planner.

4. Fiscal Policy with Education Sector intensive in Physical capital

The aim of this first part is to prove the existence of the unique stationary equilibrium when the economic path is intensive in physical capital.

According to Rebelo (1991), good production function and education sector function used are respectively expressed such that:

\[ Y = B(vK)^\mu (uh)^{1-\mu} \]
\[ h = A[(1 - v)K]^\alpha [(1 - u)h]^{1-\alpha} \]

The representative consumer spends a fraction of time, 1-\( u \) in the education sector to accumulate human capital and a fraction 1-\( v \) to good production sector. Therefore, physical capital accumulation law of motion is expressed such that, \( K = I_k - \delta_k K \)

If \( \alpha \neq \mu \) then human capital accumulation law of motion technology differs to which one used for production goods

If \( \alpha = \mu \) then the model use one sector and then, \( u=v \) both in good production and education sector
If \( \alpha \leq \mu \) then the education sector is intensive in human capital and the good production sector is intensive in physical capital.

The dynamic equilibrium of this model was studied by Bond et al (1996); Mulligan-Sala-i-Martin (1993) and find that the value of \( \alpha \) compare to the value of \( \mu \), affects transitional dynamics of the model. Ortigueira (1998) studies the imapct of fiscal policy in endogenous growth model when \( \alpha \leq \mu \) only meaning that the education sector is intensive in human capital accumulation.

**Proposition 3**: in the economy intensive in physical capital, the optimal fiscal policy is defined by the following expressions

\[
1 - \tau_k = \left[ \frac{\delta_k + \rho + \sigma g}{A \Delta_k} \right]^{\mu/3(3-\mu)}
\]  
(23)

\[
1 - \tau_h = \left[ \frac{\delta_h + \rho + \sigma g}{(1-\mu)A \Delta_k} \right]^{\mu/3(3-\mu)}
\]  
(24)

Where \( \Delta_k = \left[ \frac{1}{1-v} \left( \frac{g + \delta_h}{A} \right)^{1/\alpha} \right] \)

(See the appendix for proof)

**Proposition 4**: fiscal policy of the economy intensive in physical capital given by proposition 3 leads to a unique optimal sustainable path given by (25)-(29) i.e

\[
g = \frac{A - (\rho + \delta_h)}{\sigma} \]  
(25)

\[
u = \frac{1}{(1-\tau_k)^{\mu-1}(1-\mu)} \left[ \left( \frac{g + \delta_h}{A(1-v)^{\alpha}} \right)^{1/\alpha} \left( \frac{Bv^\mu}{\rho + \sigma g + \delta_k} \right)^{1/\mu-1} \right] \]  
(26)

\[
k = (1-\tau_k)^{2-\mu/1-\mu} \left[ \left( \frac{\rho + \sigma g + \delta_k}{Bv^\mu} \right)^{1/\mu(1-\mu)} \left( \frac{g + \delta_h}{A(1-v)^{\alpha}} \right)^{1/\alpha} \right] \]  
(27)
\[ z = (1 - \tau_h) \frac{A}{1 - \mu} - (\delta_k + g) g \]  

(See the appendix for proof)

Now the production sector time devoted is taxed, thus education remaining time too due to parents excess finance support to education through the wage rate income. Then consumption demand decreases as well as time spent in education sector, thus development stagnates since the balanced growth path includes income shortness.

**Proposition 5**: there exist a threshold, \( \Pi \) where fiscal policy is detrimental for economic growth defined by:

\[ \Pi = \left[ \rho + (1 - \sigma)(\delta_h - A)(1 - \nu) \right] \left[ g + \delta_h(\rho + \sigma g + \delta_k) \right]^{1/1-\mu} \]  

Indeed, when fiscal policy is below \( \Pi \), fiscal policy is profitable for development; when it equals \( \Pi \), then the balanced growth path is reached and finally when fiscal policy crossed the threshold, \( \Pi \), the policy of taxation is detrimental for economic growth, thus for development purpose. (see figure 1)
5. Conclusion

The aim of this analysis was to see in how far, fiscal policy may yield development. We find that, as long as the critical level of taxes is not achieved yet, it can be positive for growth and development economics. But after the threshold crossed, the economy stagnates and yields development retard. Consequently, human capital increase through fiscal policy is limited.

References


Appendix

Proof of proposition 1

We fix first, the system to physical capital measure such that...
\[ \rho + \sigma g = (1 - \tau_k) \mu BK^{\mu^{-1}}(uh)^{1-\mu} - \delta_k \]
\[ (1 - \tau_k) \mu BK^{\mu^{-1}}(uh)^{1-\mu} - \delta_k = A - \delta_h \]
\[ \frac{c}{K} + g = BK^{\mu^{-1}}(uh)^{1-\mu} - \delta_k \]
\[ g = A(1-u) - \delta_h \]

Equations can be numbered (1)-(4)

Doing (1)=(2), we determinate the growth rate expression such that

\[ g = \frac{1}{\sigma} [A - (\rho + \delta_h)] \] \hspace{1cm} A

Taking account of the previous equation added to equation (4), we obtain time spent in the production sector, \( u \) such that

\[ u = \frac{\rho + (1-\sigma)(\delta_h - A)}{A\sigma} \] \hspace{1cm} B

From equation (2), we obtain the expression of \( k = K/h \) and using it as well as equation (3), we obtain \( z = c/K \) respectively expressed such that

\[ k = (1 - \tau_k)^{1/\mu} \phi_k \] \hspace{1cm} C
\[ z = \frac{B}{1 - \tau_k} - \phi_{iz} \] \hspace{1cm} D

Where

\[ \phi_k = \frac{1}{u} \left( \frac{\mu B}{A - \delta_h + \delta_k} \right)^{1/\mu} \]
\[ \phi_{iz} = (\delta_k + g) \]

We now fix the system to human capital measure such that
\[ \rho + \sigma g = (1 - \tau_h)(1 - \mu)BK^\mu(uh)^{-\mu} - \delta_h \]
\[ (1 - \tau_h)(1 - \mu)BK^\mu(uh)^{-\mu} - \delta_h = A - \delta_h \]
\[ \frac{c}{K} + g = BK^{-1}(uh)^{1-\mu} - \delta_k \]
\[ g = A(1-u) - \delta_h \]

The goal is to determinate the same variables as before. We find that \( g \) and \( u \) remain the same but \( k \) and \( z \) change and become expressed such that

\[ k = \left( \frac{1}{1 - \tau_h} \right)^{1/\mu} \phi_{zk} \quad \text{C'} \]
\[ z = (1 - \tau_h)^{\mu/1-\mu} \phi_{zc} - (g + \delta_k) \quad \text{D'} \]

Where

\[ \phi_{zk} = \left[ \frac{\rho + \delta_h + \sigma g}{B(1 - \mu)} \right]^{1/\mu} u \]
\[ \phi_{zc} = \left[ \phi_{zk} \right]^{1-\mu} \]

We can see that, (C) and (D) are respectively increasing and decreasing functions of fiscal policy when \( z = k \), therefore, the equilibrium exist such that taking the logarithme of (C) = (D) and making approximation around 0 of \( 1 - \frac{\phi_{zk}}{(1 - \tau_h)^{\mu/1-\mu} \phi_{zk}} \approx 1 \)

We obtain \( 1 - \tau_k = \left( \frac{B}{\phi_{zk}} \right)^{1-\mu} \)

Now we can see that, (C') and (D') are increasing and decreasing functions of fiscal policy when \( z = k \), therefore, the equilibrium exist such that solving (C')=(D') and taking the approximation around 0 of \( 1 - \frac{g + \delta_h}{\phi_{zc}(1 - \tau_h)^{\mu/1-\mu}} \approx 1 \) it yields
Indeed, fiscal policy is defined such that

\[ 1 - \tau_h = \left[ \frac{\phi_{2k}}{\phi_{2z}} \right]^{\mu(1-\mu)/1-\mu+\mu^2} \]

\[ 1 - \tau_k = \left( \frac{B}{\phi_{1k}} \right)^{1-\mu} \tag{17} \]

\[ 1 - \tau_h = \left[ \frac{\phi_{2k}}{\phi_{2z}} \right]^{\mu(1-\mu)/1-\mu+\mu^2} \tag{18} \]

**Proof of proposition 2**

In order to determinate the associated optimal growth path, we generalize the equations system such that

\[ \rho + \sigma g = (1 - \tau_k) \mu BK^{-1} (uh)^{1-\mu} - \delta_k \]
\[ (1 - \tau_h)(1 - \mu) BK^{\mu} (uh)^{1-\mu} - \delta_h = A - \delta_h \]
\[ \frac{c}{K} + g = BK^{\mu-1}(uh)^{1-\mu} - \delta_k \]
\[ g = A(1-u) - \delta_h \]

Equalizing the first and the second equation, we determination the economic growth rate. Using the equation found added to equation (4), we obtain time spent in production sector. Using separatly equations (1), (2) and (3) we determinate \( z \) as well as \( k \) final expressions. Therefore, the optimal balanced growth path is expressed such that:

\[ g = \frac{A - (\rho + \delta_h)}{\sigma} \tag{19} \]
\[ u = \frac{\rho + (1 - \sigma)(\delta_h - A)}{\sigma A} \tag{20} \]
\[ k = (1 - \tau_k)^{1/\mu} \phi_k \quad (21) \]
\[ z = B / (1 - \tau_k) \phi_z^1 - \phi_z^2 \quad (22) \]

Where
\[ \phi_k = \left( \frac{\mu B}{\rho + \sigma g + \delta_k} \right)^{1/\mu} u \]
\[ \phi_z^1 = \left( \frac{\mu B}{\rho + \sigma g + \delta_k} \right)^{-1} \]
\[ \phi_z^2 = g + \delta_k \]

**Proof of proposition3 : fiscal policy determination : second case**

Since we now have
\[ Y = B(vK)^\nu (uh)^{1-\mu} \]
\[ h = A[(1 - v)K]^{\nu} [(1 - \mu)h]^{1-\mu} - \delta_h h(t) \]

Indeed, \( r(t) \) and \( w(t) \) became expressed such that
\[ r(t) = B \mu \nu^\nu u^{1-\mu} k^{\mu-1} \]
\[ w(t) = B(1 - \mu) \nu^\nu u^{1-\mu} k^{\mu} \]

The system became for the first case of physical capital term expressed such that
\[
\begin{align*}
\rho + \sigma g &= (1 - \tau_k)B \mu v^\mu u^{1-\mu} k^{\mu-1} - \delta_k \\
(1 - \tau_k)B \mu v^\mu u^{1-\mu} k^{\mu-1} - \delta_k &= A - \delta_h \\
c + g &= B v^\mu k^{\mu-1} u^{1-\mu} - \delta_k \\
g &= A(1-u)^{1-\alpha} (1-v)^{\alpha} k^\alpha - \delta_h
\end{align*}
\]

**Assumption 1**: \((1-u)^{1-\alpha} \approx 1\)

In terms of physical capital

Doing the same calculus than in the previous case where A1 is true, using (1) yields an expression of \(k/u\) and the same thing happen when (4) is used. Indeed, using one of the two expression in (3), it yields the expression of \(z=c/K\) such that

\[
z = \frac{1}{1-\tau_k} \left( \rho + \sigma g + \delta_k \right) - \left( \delta_h + g \right)
\]

Using the second \(k/u\) expression found in equation (1) added with A1, we find the expression of \(u\) such that

\[
u = \left(1 - \tau_k\right) \left[ \frac{g + \delta_h}{A(1-v)^\alpha} \left( \rho + \sigma g + \delta_k \right)^{\alpha/(1-\mu)} \right]
\]

Now replacing \(u\) inside one of the expression \(k/u\) found earlier, we determinate the expression of \(k\) such that

\[
k = \left(1 - \tau_k\right)^{2-\mu/1-\mu} \left[ \frac{g + \delta_h}{A(1-v)^\alpha} \left( \rho + \sigma g + \delta_k \right)^{\alpha/(1-\mu)} \right]
\]

Like before \(g\) remains the same when we make (1) equals (2)

\[
g = \frac{A - \left( \rho + \delta_h \right)}{\sigma}
\]
In terms of human capital, we have the following system equations

\[
\begin{align*}
\rho + \sigma g &= (1 - \tau_h)B\mu^\mu u^{1-\mu}k^{\mu-1} - \delta_h \\
(1 - \tau_h)B\mu^\mu u^{1-\mu}k^{\mu-1} - \delta_h &= A - \delta_h \\
c/K + g &= Bv^\mu k^{\mu-1}u^{1-\mu} - \delta_h \\
g &= A(1-u)^{1-\alpha}(1-v)^\alpha k^\alpha - \delta_h
\end{align*}
\]

The temporary optimal growth path is now found exactly like before which yields

\[
\begin{align*}
z &= \left( \frac{1}{1 - \tau_h} \right) \left[ \frac{\rho + \sigma g + \delta_h}{1 - \mu} \right] - (g + \delta_h) & \text{A'} \\
\bar{u} &= (1 - \tau_h)^{1/\mu} \Delta_u & \text{B'}
\end{align*}
\]

Where \( \Delta_u = \left( \frac{g + \delta_h}{A(1 - v)^\alpha} \right)^{1/\alpha} \left[ \frac{B(1 - \mu)v^\mu}{\rho + \sigma g + \delta_h} \right]^{1/\mu-1} \)

\[
k = (1 - \tau_h)^{2/\mu} \left[ \left( \frac{1}{1 - v} \right) \left( \frac{g + \delta_h}{A} \right) \right]^{1/\alpha} & \text{C'}
\]

\[
g = A - \frac{(\rho + \delta_h)}{\sigma} & \text{D'}
\]

Finally, when (A)=(C) and (A')=(C') optimal fiscal policy is now determinated such that
1 - τ_κ = \left[ \frac{\delta_κ + \rho + \sigma g}{\Delta_κ} \right]^{-(\mu/3 - 2\mu)} \tag{23}

Where \Delta_κ = \left[ \frac{1}{1 - \nu} \left( g + \delta_h \right)^{1/\alpha} \right]

1 - τ_κ = \left[ \frac{\delta_h + \rho + \sigma g}{1 - \mu \Delta_κ} \right]^{-(\mu/3 - \mu)} \tag{24}

Proof of proposition 4: the unique optimal growth path of the second case

Generalizing the equations system such that

ρ + σg = (1 - τ_κ)Bv^κu^{1-\mu}k^{\mu-1} - δ_κ
(1 - τ_κ)B(1 - μ)v^κu^{1-\mu}k^{\mu-1} - δ_κ = A - δ_h

\frac{c}{K} + g = Bv^μk^{\mu-1}u^{1-\mu} - δ_κ
g = A(1 - u)^{1-\alpha} (1 - v)^α k^α - δ_h

It yields like in the previous case that

\begin{align*}
\frac{1}{(1 - τ_κ)^{1/(1-\mu)}} & \left[ \left( \frac{g + \delta_h}{A(1 - v)^α} \right)^{1/\alpha} \left( \frac{Bv^μ}{\rho + σg + δ_κ} \right)^{1/1-\mu} \right] \tag{26}

k & = (1 - τ_κ)^{1/(1-\mu)} \left[ \left( \frac{\rho + σg + δ_κ}{Bv^μ} \right)^{1/(1-\mu)} \left( \frac{g + \delta_h}{A(1 - v)^α} \right)^{1/\alpha} \right] \tag{27}

z & = (1 - τ_κ) \frac{A}{1 - μ} (δ_κ + g) \tag{28}
\end{align*}
Proof of proposition 5: FISCAL POLICY THRESHOLD DETERMINATION

Setting

\[
\begin{align*}
  u &= \frac{\rho + (1 - \sigma)(\delta_h - A)}{\sigma A} = u = \frac{1}{(1 - \tau_k)^{1/(1 - \mu)}} \left[ \left( \frac{g + \delta_h}{A(1 - \nu)} \right)^{1/\alpha} \left( \frac{Bv^\mu}{\rho + \sigma g + \delta_k} \right)^{1/\mu - 1} \right] \\

\end{align*}
\]

It yield the existence of a bound in fiscal policy terms such that \( \Pi \) such that

\[
\Pi = \frac{\left[ \rho + (1 - \sigma)(\delta_h - A) \right](1 - \nu)^\alpha}{(g + \delta_h)(\rho + \sigma g + \delta_k)^{1/1 - \mu}} \quad (29)
\]